SL Paper 2

Give your answers to parts (a) to (e) to the nearest dollar.

On Hugh's 18th birthday his parents gave him options of how he might receive his monthly allowance for the next two years.

Option A $60 each month for two years

Option B $10 in the first month, $15 in the second month, $20 in the third month, increasing by $5 each month for two years

Option C $15 in the first month and increasing by 10% each month for two years

Option D Investing $1500 at a bank at the beginning of the first year, with an interest rate of 6% per annum, **compounded monthly**.

Hugh does not spend any of his allowance during the two year period.

a. If Hugh chooses **Option A**, calculate the total value of his allowance at the end of the two year period. [2]

b. If Hugh chooses **Option B**, calculate [5]

(i) the amount of money he will receive in the 17th month;

(ii) the total value of his allowance at the end of the two year period.

c. If Hugh chooses **Option C**, calculate [5]

(i) the amount of money Hugh would receive in the 13th month;

(ii) the total value of his allowance at the end of the two year period.

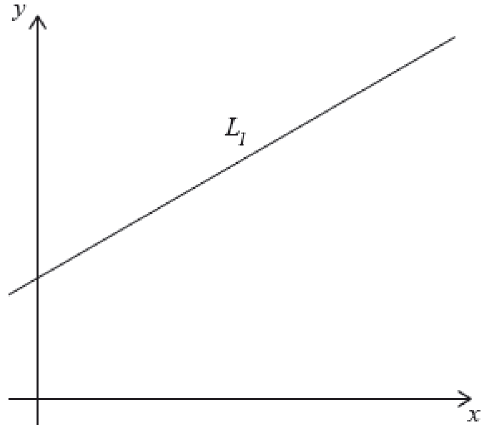
d. If Hugh chooses **Option D**, calculate the total value of his allowance at the end of the two year period. [3]

e. State which of the options, A, B, C or D, Hugh should choose to give him the greatest total value of his allowance at the end of the two year period. [1]

f. Another bank guarantees Hugh an amount of $\$1750$ after two years of investment if he invests $1500 at this bank. The interest is **compounded annually**. [3]

Calculate the interest rate per annum offered by the bank.

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1,\ 4)$.

The point C has coordinates $(5,\ 12)$. M is the midpoint of AC.

The straight line, L_2, is perpendicular to AC and passes through M.

The point D is the intersection of L_1 and L_2.

The length of MD is $\frac{\sqrt{45}}{2}$.

The point B is such that ABCD is a rhombus.

a. Show that A lies on L_1. [2]

b. Find the coordinates of M. [2]

c. Find the length of AC. [2]

d. Show that the equation of L_2 is $2y + x - 19 = 0$. [5]

e. Find the coordinates of D. [2]

f. Write down the length of MD correct to five significant figures. [1]

g. Find the area of ABCD. [3]

The natural numbers: 1, 2, 3, 4, 5… form an arithmetic sequence.

A geometric progression G_1 has 1 as its first term and 3 as its common ratio.

i.a. State the values of u_1 and d for this sequence. [2]

i.b. Use an appropriate formula to show that the sum of the natural numbers from 1 to n is given by $\frac{1}{2}n(n+1)$. [2]

i.c. Calculate the sum of the natural numbers from 1 to 200. [2]

ii.a. The sum of the first n terms of G_1 is 29 524. Find n. [3]

ii.b. A second geometric progression G_2 has the form $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \dots$ [1]

ii.c. Calculate the sum of the first 10 terms of G_2. [2]

ii.d. Explain why the sum of the first 1000 terms of G_2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures. [1]

ii.e. Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence $2, 3\frac{1}{3}, 9\frac{1}{9}, 27\frac{1}{27} \dots$ [3]

Give your answer **correct to one decimal place**.

Daniel wants to invest \$25 000 for a total of three years. There are two investment options.

Option One pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

Option Two pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \quad n = 1, 2, 3, \dots$$

A.a. Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**. [8]

A.b. Determine Daniel's best investment option. [1]

B.a. Calculate u_1, the first term in the sequence. [2]

B.b. Show that the common difference is 7. [2]

B.c. S_n is the sum of the first n terms of the sequence. [3]

Find an expression for S_n. Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants.

B.d. The first term, v_1, of a geometric sequence is 20 and its fourth term v_4 is 67.5. [2]

Show that the common ratio, r, of the geometric sequence is 1.5.

B.e. T_n is the sum of the first n terms of the geometric sequence. [2]

Calculate T_7, the sum of the first seven terms of the geometric sequence.

B.f. T_n is the sum of the first n terms of the geometric sequence. [2]

Use your graphic display calculator to find the smallest value of n for which $T_n > S_n$.

Give all answers in this question correct to two decimal places.

Part A

Estela lives in Brazil and wishes to exchange 4000 BRL (Brazil reals) for GBP (British pounds). The exchange rate is 1.00 BRL = 0.3071 GBP. The bank charges 3 % commission on the amount in BRL.

Give all answers in this question correct to two decimal places.

Part B

Daniel invests $1000 in an account that offers a nominal annual interest rate of 3.5 % **compounded half yearly**.

A.a Find, **in BRL**, the amount of money Estela has after commission. [2]

A.b Find, in GBP, the amount of money Estela receives. [2]

A.c After her trip to the United Kingdom Estela has 400 GBP left. At the airport she changes this money back into BRL. The exchange rate is now [2]

1.00 BRL = 0.3125 GBP.

Find, in BRL, the amount of money that Estela should receive.

A.d Estela actually receives 1216.80 BRL after commission. [1]

Find, in BRL, the commission charged to Estela.

A.e The commission rate is t % . Find the value of t. [2]

B.a Show that after three years Daniel will have $1109.70 in his account, correct to two decimal places. [3]

B.b Write down the interest Daniel receives after three years. [1]

In the diagram below A, B and C represent three villages and the line segments AB, BC and CA represent the roads joining them. The lengths of AC and CB are 10 km and 8 km respectively and the size of the angle between them is 150°.

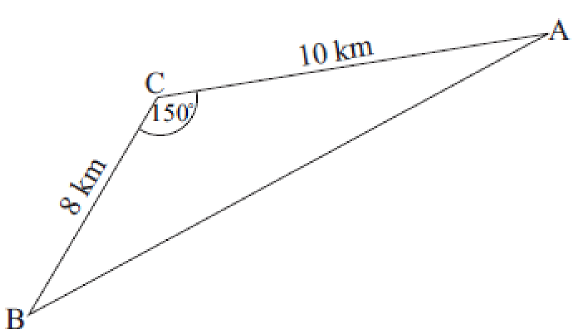


diagram not to scale

a. Find the length of the road AB. [3]

b. Find the size of the angle CAB. [3]

c. Village D is halfway between A and B. A new road perpendicular to AB and passing through D is built. Let T be the point where this road cuts AC. This information is shown in the diagram below. [1]

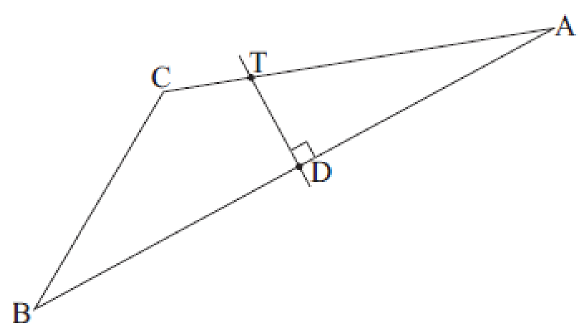


diagram not to scale

Write down the distance from A to D.

d. Show that the distance from D to T is 2.06 km correct to three significant figures. [2]

e. A bus starts and ends its journey at A taking the route AD to DT to TA. [3]

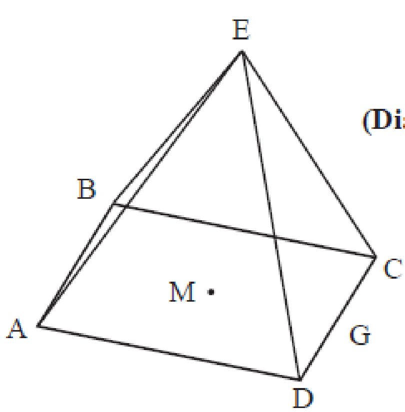
Find the total distance for this journey.

f. The average speed of the bus while it is moving on the road is 70 km h^{-1}. The bus stops for **5 minutes** at each of D and T . [4]

Estimate the time taken by the bus to complete its journey. Give your answer correct to the nearest minute.

Mal is shopping for a school trip. He buys 50 tins of beans and 20 packets of cereal. The total cost is 260 Australian dollars (AUD).

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



(Diagram not to scale)

i.a. Write down an equation showing this information, taking b to be the cost of one tin of beans and c to be the cost of one packet of cereal in AUD. [1]

i.b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. [1]

Write down another equation to represent this information.

i.c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. [2]

Find the cost of one tin of beans.

i.d.(i) Sketch the graphs of the two equations from parts (a) and (b). [4]

(ii) Write down the coordinates of the point of intersection of the two graphs.

ii.a. Using the letters on the diagram draw a triangle showing the position of a 70° angle. [1]

ii.b. Show that the height of the pyramid is 13.7 cm, to 3 significant figures. [2]

ii.c. Calculate [4]

(i) the length of EG;

(ii) the size of angle DEC.

ii.d. Find the total surface area of the pyramid. [2]

ii.e. Find the volume of the pyramid. [2]

Rosa joins a club to prepare to run a marathon. During the first training session Rosa runs a distance of 3000 metres. Each training session she increases the distance she runs by 400 metres.

A marathon is 42.195 kilometres.

In the kth training session Rosa will run further than a marathon for the first time.

Carlos joins the club to lose weight. He runs 7500 metres during the first month. The distance he runs increases by 20% each **month**.

a.i. Write down the distance Rosa runs in the third training session; [1]

a.ii. Write down the distance Rosa runs in the nth training session. [2]

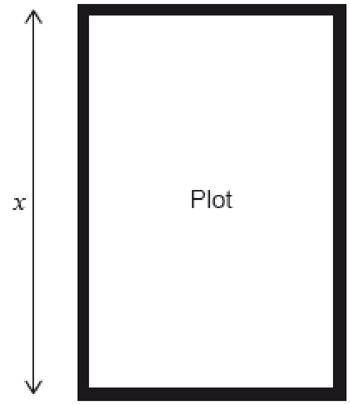
b. Find the value of k. [2]

c. Calculate the total distance, in **kilometres**, Rosa runs in the first 50 training sessions. [4]

d. Find the distance Carlos runs in the fifth month of training. [3]

e. Calculate the total distance Carlos runs in the first year. [3]

Violeta plans to grow flowers in a rectangular plot. She places a fence to mark out the perimeter of the plot and uses 200 metres of fence. The length of the plot is x metres.



Violeta places the fence so that the area of the plot is maximized.

By selling her flowers, Violeta earns 2 Bulgarian Levs (BGN) per square metre of the plot.

Violeta wants to invest her 5000 BGN.

A bank offers a nominal annual interest rate of 4%, compounded **half-yearly**.

Another bank offers an interest rate of r% compounded **annually**, that would allow her to double her money in 12 years.

a. Show that the width of the plot, in metres, is given by $100 - x$. [1]

b. Write down the area of the plot in terms of x. [1]

c. Find the value of x that maximizes the area of the plot. [2]

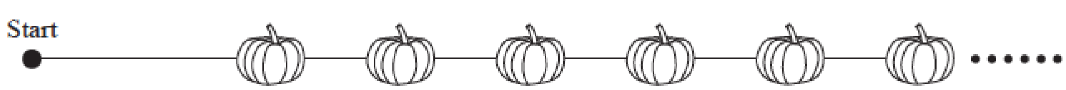
d. Show that Violeta earns 5000 BGN from selling the flowers grown on the plot. [2]

e.i. Find the amount of money that Violeta would have after 6 years. Give your answer correct to two decimal places. [3]

e.ii. Find how long it would take for the interest earned to be 2000 BGN. [3]

f. Find the lowest possible value for r. [2]

In a game, *n* small pumpkins are placed 1 metre apart in a straight line. Players start 3 metres before the first pumpkin.



Each player **collects** a single pumpkin by picking it up and bringing it back to the start. The nearest pumpkin is collected first. The player then collects the next nearest pumpkin and the game continues in this way until the signal is given for the end.

Sirma runs to get each pumpkin and brings it back to the start.

a. Write down the distance, a_1, in metres that she has to run in order to **collect** the first pumpkin. [1]

b. The distances she runs to **collect** each pumpkin form a sequence $a_1,\ a_2,\ a_3, \ldots$. [2]

(i) Find a_2.

(ii) Find a_3.

c. Write down the common difference, d, of the sequence. [1]

d. The final pumpkin Sirma **collected** was 24 metres from the start. [5]

(i) Find the total number of pumpkins that Sirma **collected**.

(ii) Find the total distance that Sirma ran to **collect** these pumpkins.

e. Peter also plays the game. When the signal is given for the end of the game he has run 940 metres. [3]

Calculate the total number of pumpkins that Peter **collected**.

f. Peter also plays the game. When the signal is given for the end of the game he has run 940 metres. [2]

Calculate Peter's distance from the start when the signal is given.

Part A

The Green Park Amphitheatre was built in the form of a horseshoe and has 20 rows. The number of seats in each row increase by a fixed amount, d, compared to the number of seats in the previous row. The number of seats in the sixth row, u_6, is 100, and the number of seats in the tenth row, u_{10}, is 124. u_1 represents the number of seats in the first row.

Part B

Frank is at the amphitheatre and receives a text message at 12:00. Five minutes later he forwards the text message to three people. Five minutes later, those three people forward the text message to three new people. **Assume this pattern continues and each time the text message is sent to people who have not received it before.**

The number of new people who receive the text message forms a geometric sequence

$$1\ ,\ 3\ ,\ \ldots$$

A.a(i) Write an equation for u_6 in terms of d and u_1. [2]

(ii) Write an equation for u_{10} in terms of d and u_1.

A.b Write down the value of [2]

(i) d ;

(ii) u_1 .

A.c Find the **total** number of seats in the amphitheatre. [3]

A.d A few years later, a **second** level was added to increase the amphitheatre's capacity by another 1600 seats. Each row has four more seats than the previous row. The first row on this level has 70 seats. [4]

Find the number of rows on the second level of the amphitheatre.

B.a Write down the next two terms of this geometric sequence. [1]

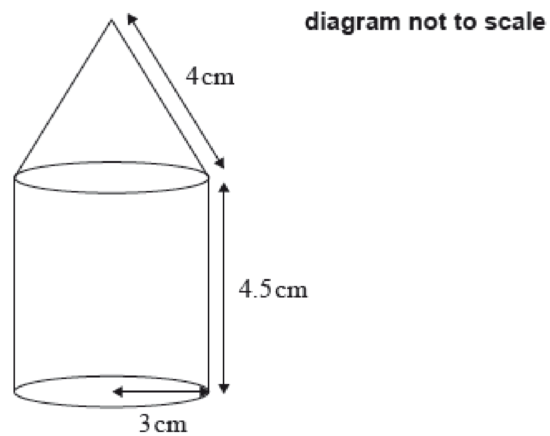
B.b Write down the common ratio of this geometric sequence. [1]

B.c Calculate the number of people who will receive the text message at 12:30. [2]

B.d Calculate the **total** number of people who will have received the text message by 12:30. [2]

B.e Calculate the exact time at which a total of 29 524 people will have received the text message. [3]

The following diagram shows a perfume bottle made up of a cylinder and a cone.



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

a. (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures. [6]

(ii) Calculate the volume of the perfume bottle.

b. The bottle contains $125\ \mathrm{cm}^3$ of perfume. The bottle is **not** full and all of the perfume is in the cylinder part. [2]

Find the height of the perfume in the bottle.

c. Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle. [4]

Find the **total** surface area of the perfume bottle.

d. Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of $7\ \mathrm{cm}^2$. [4]

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**.

e. Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR. [2]

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places**.

A geometric sequence has second term 12 and fifth term 324.

Consider the following propositions

p: The number is a multiple of five.

q: The number is even.

r: The number ends in zero.

i, a. Calculate the value of the common ratio. [4]

i, b. Calculate the 10$^{\text{th}}$ term of this sequence. [3]

i, c. The k^{th} term is the first term which is greater than 2000. Find the value of k. [3]

ii, a. Write in words $(q \wedge \neg r) \Rightarrow \neg p$. [3]

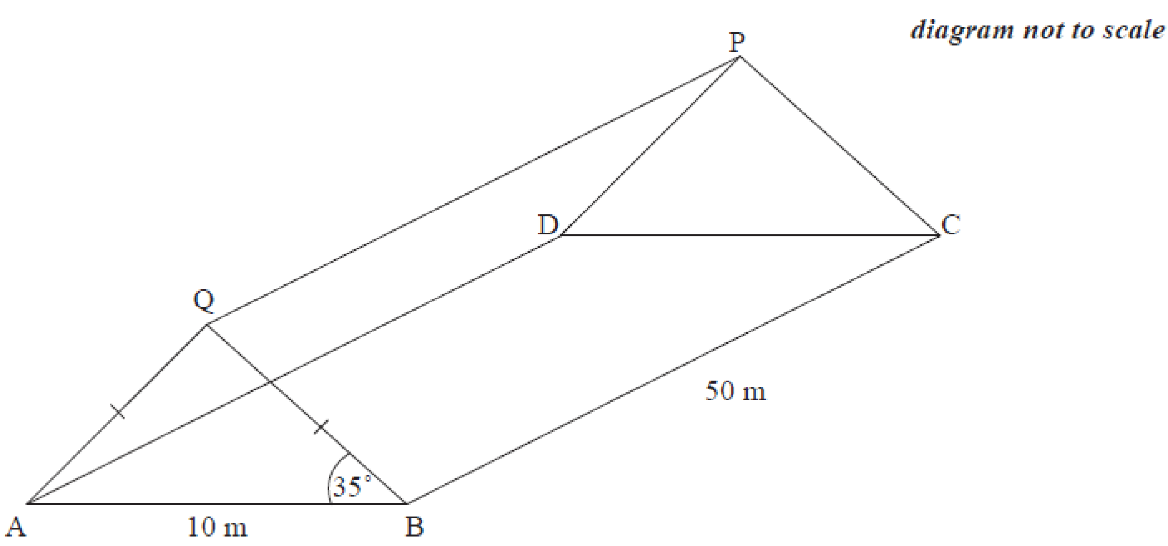
ii, b, i. Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero". [4]

Write this statement in symbolic form.

ii, b, ii. Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero". [2]

Write the contrapositive of this statement in symbolic form.

A greenhouse ABCDPQ is constructed on a rectangular concrete base ABCD and is made of glass. Its shape is a right prism, with cross section, ABQ, an isosceles triangle. The length of BC is 50 m, the length of AB is 10 m and the size of angle QBA is 35°.



a. Write down the size of angle AQB. [1]

b. Calculate the length of AQ. [3]

c. Calculate the length of AC. [2]

d. Show that the length of CQ is 50.37 m, correct to 4 significant figures. [2]

e. Find the size of the angle AQC. [3]

f. Calculate the total area of the glass needed to construct [5]

(i) the two rectangular faces of the greenhouse;

(ii) the two triangular faces of the greenhouse.

g. The cost of one square metre of glass used to construct the greenhouse is 4.80 USD. [3]

Calculate the cost of glass to make the greenhouse. Give your answer correct to the nearest 100 USD.

Leanne goes fishing at her favourite pond. The pond contains four different types of fish: bream, flathead, whiting and salmon. The fish are either undersized or normal. This information is shown in the table below.

Size / Type of fish	Bream	Flathead	Whiting	Salmon	Total
Undersized	3	12	18	9	42
Normal	0	11	24	13	48
Total	3	23	42	22	

a. Write down the total number of fish in the pond. [1]

b. Leanne catches a fish. [7]

Find the probability that she

(i) catches an undersized bream;

(ii) catches either a flathead or an undersized fish or both;

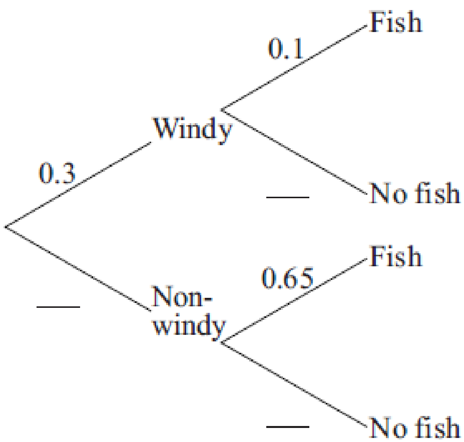
(iii) does not catch an undersized whiting;

(iv) catches a whiting given that the fish was normal.

c. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Copy and complete the probability tree diagram below.



d. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [2]

The probability that it will be windy on a particular day is 0.3.

Calculate the probability that it is windy and Leanne catches a fish on a particular day.

e. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Calculate the probability that Leanne catches a fish on a particular day.

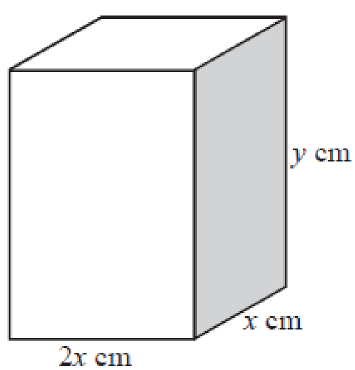
f. Use your answer to part (e) to calculate the probability that Leanne catches a fish on two consecutive days. [2]

g. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Given that Leanne catches a fish on a particular day, calculate the probability that the day was windy.

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



i.a. Factorise $3x^2 + 13x - 10$. [2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$. [2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$. [2]

ii.b. Find an expression for y in terms of x. [2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. [2]

ii.d. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

ii.e. (i) Hence find the value of x and of y required to make the volume of the box a maximum. [5]

(ii) Calculate the maximum volume.

Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.

Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

a. Find the amount of money that Ruby will have in the bank after 3 years. [2]

b. Show that Ying will have 7545 USD in the bank at the end of 10 years. [3]

c. Find the number of complete years it will take for Ying's investment to first exceed 6500 USD. [3]

d. Find the number of complete years it will take for Ying's investment to exceed Ruby's investment. [3]

e. Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 Euros. The bank charges 1.8 % commission. [4]

Calculate the amount of money Ruby will invest in the Italian bank after commission.

f. Ruby returns to the USA for a short holiday. She converts 800 Euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 Euro = 1.29 USD. [5]

Calculate the percentage commission Ruby is charged by the bank.

Give all answers in this question correct to the ***nearest*** *dollar.*

Clara wants to buy some land. She can choose between two different payment options. Both options require her to pay for the land in **20** monthly installments.

Option 1: The first installment is \$2500. Each installment is \$200 more than the one before.

Option 2: The first installment is \$2000. Each installment is 8% more than the one before.

a. If Clara chooses option 1, [7]

(i) write down the values of the second and third installments;

(ii) calculate the value of the final installment;

(iii) show that the **total amount** that Clara would pay for the land is $\$88000$.

b. If Clara chooses option 2, [4]

(i) find the value of the second installment;

(ii) show that the value of the fifth installment is $\$2721$.

c. The price of the land is $\$80000$. In option 1 her total repayments are $\$88000$ over the **20** months. Find the annual rate of simple interest which gives this total. [4]

d. Clara knows that the **total amount** she would pay for the land is not the same for both options. She wants to spend the least amount of money. Find how much she will save by choosing the cheaper option. [4]

A geometric sequence has 1024 as its first term and 128 as its fourth term.

Consider the arithmetic sequence $1,\ 4,\ 7,\ 10,\ 13,\ \ldots$

A.a Show that the common ratio is $\frac{1}{2}$. [2]

A.b Find the value of the eleventh term. [2]

A.c Find the sum of the first eight terms. [3]

A.d Find the number of terms in the sequence for which the **sum** first exceeds 2047.968. [3]

B.a Find the value of the eleventh term. [2]

B.b The sum of the first n terms of this sequence is $\frac{n}{2}(3n-1)$. [6]

(i) Find the sum of the first 100 terms in this arithmetic sequence.

(ii) The sum of the first n terms is 477.

(a) Show that $3n^2 - n - 954 = 0$.

(b) Using your graphic display calculator or otherwise, find the number of terms, n .

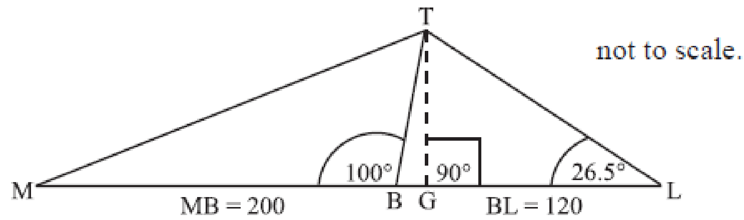
Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

An old tower (BT) leans at $10°$ away from the vertical (represented by line TG).

The base of the tower is at B so that $\mathrm{M\hat{B}T} = 100°$.

Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.

He measures the angle between the ground and the top of the tower T to be $\mathrm{B\hat{L}T} = 26.5°$.



i.a. Calculate the volume of the cylinder **in cm³**. Give your answer correct to **two** decimal places. [3]

i.b. The cylinder is used for storing tennis balls. Each ball has a **radius** of 3.25 cm. [1]

Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

i.c. (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls. [4]

(ii) Convert your answer to (c) (i) into cubic metres.

ii.a. (i) Find the value of angle $\mathrm{B\hat{T}L}$. [5]

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.

ii.b. Calculate the vertical height TG of the top of the tower. [2]

ii.c. Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T. [3]

Consider the sequence $u_1,\ u_2,\ u_3,\ \ldots,\ \mathrm{u}_n,\ \ldots$ where

$$u_1 = 600,\ u_2 = 617,\ u_3 = 634,\ u_4 = 651.$$

The sequence continues in the same manner.

a. Find the value of u_{20}. [3]

b. Find the sum of the first 10 terms of the sequence. [3]

c. Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \ldots$ where [3]

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.

Find the exact value of v_{10}.

d. Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \ldots$ where [3]

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.

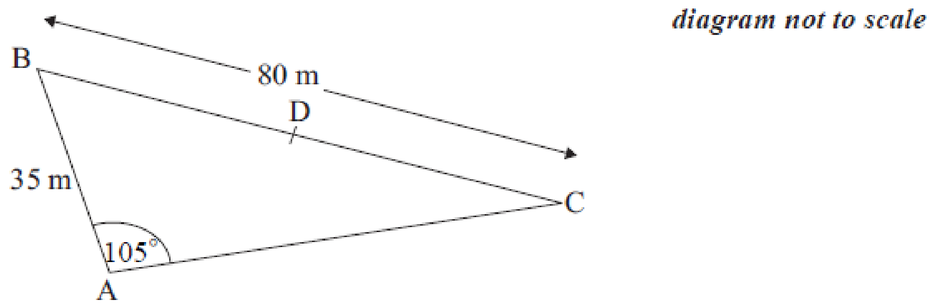
Find the sum of the first 8 terms of this sequence.

e. k is the smallest value of n for which v_n is greater than u_n. [3]

Calculate the value of k.

A farmer has a triangular field, ABC, as shown in the diagram.

AB = 35 m, BC = 80 m and BÂC = 105°, and D is the midpoint of BC.



a. Find the size of BĈA. [3]

b. Calculate the length of AD. [5]

c. The farmer wants to build a fence around ABD. [2]

Calculate the total length of the fence.

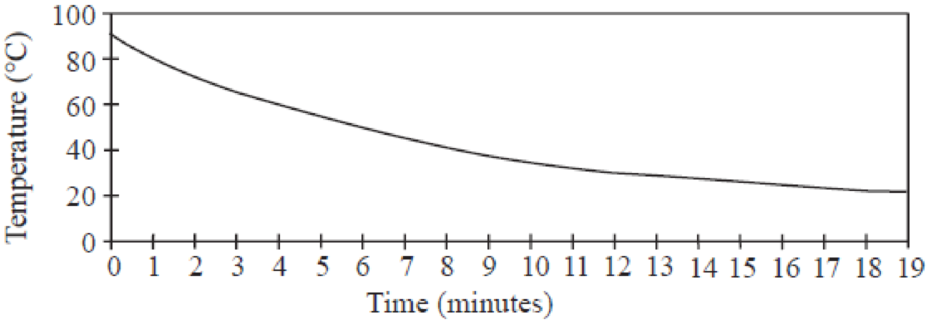
d. The farmer wants to build a fence around ABD. [2]

The farmer pays 802.50 USD for the fence. Find the cost per metre.

e. Calculate the area of the triangle ABD. [3]

f. A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic metres. [3]

The following graph shows the temperature in degrees Celsius of Robert's cup of coffee, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the coffee out.



Robert, who lives in the UK, travels to Belgium. The exchange rate is 1.37 euros to one British Pound (GBP) with a commission of 3 GBP, which is subtracted before the exchange takes place. Robert gives the bank 120 GBP.

i.a. Find the initial temperature of the coffee. [1]

i.b. Write down the equation of the horizontal asymptote. [1]

i.c. Find the room temperature. [1]

i.d. Find the temperature of the coffee after 10 minutes. [1]

i.e. Find the temperature of Robert’s coffee after being heated in the microwave for 30 **seconds** after it has reached the temperature in part (d). [3]

i.f. Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C. [4]

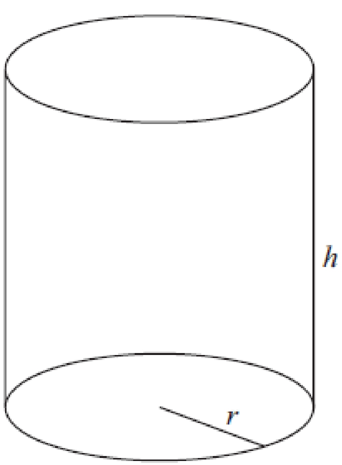
ii.a. Calculate **correct to 2 decimal places** the amount of euros he receives. [3]

ii.b. He buys 1 kilogram of Belgian chocolates at 1.35 euros per 100 g. [3]

Calculate the cost of his chocolates in GBP **correct to 2 decimal places**.

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

diagram not to scale



Nadia decides to make the radius, r , of the bin 5 cm.

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

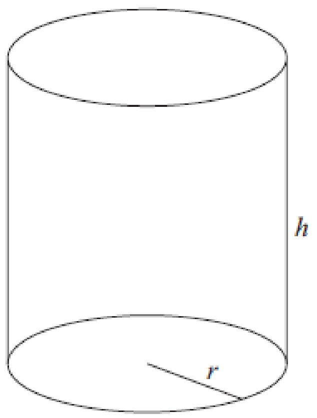


diagram not to scale

Let the radius of the base of Merryn's wastepaper bin be r , and let its height be h .

a. Calculate [7]

(i) the area of the base of the wastepaper bin;

(ii) the height, h , of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

b. State whether Nadia's design is practical. Give a reason. [2]

c. Write down an equation in h and r , using the given volume of the bin. [1]

d. Show that the total external surface area, A , of the bin is $A = \pi r^2 + \frac{16000}{r}$. [2]

e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. [5]

(ii) Calculate the value of h corresponding to this value of r .

g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. [2]

The lengths (l) in centimetres of 100 copper pipes at a local building supplier were measured. The results are listed in the table below.

Length l (cm)	**Frequency**
17.5	12
32.5	26
47.5	32
62.5	21
77.5	9

a. Write down the mode. [1]

b. Using your graphic display calculator, write down the value of [4]

(i) the mean;

(ii) the standard deviation;

(iii) the median.

c. Find the interquartile range. [2]

d. Draw a box and whisker diagram for this data, on graph paper, using a scale of $1\ \mathrm{cm}$ to represent $5\ \mathrm{cm}$. [4]

e. Sam estimated the value of the mean of the measured lengths to be $43\ \mathrm{cm}$. [2]

Find the percentage error of Sam's estimated mean.

The sum of the first n terms of an arithmetic sequence is given by $S_n = 6n + n^2$.

a. Write down the value of [2]

(i) S_1;

(ii) S_2.

b. The n^{th} term of the arithmetic sequence is given by u_n. [1]

Show that $u_2 = 9$.

c. The n^{th} term of the arithmetic sequence is given by u_n. [2]

Find the common difference of the sequence.

d. The n^{th} term of the arithmetic sequence is given by u_n. [2]

Find u_{10}.

e. The n^{th} term of the arithmetic sequence is given by u_n. [3]

Find the lowest value of n for which u_n is greater than 1000.

f. The n^{th} term of the arithmetic sequence is given by u_n. [2]

There is a value of n for which

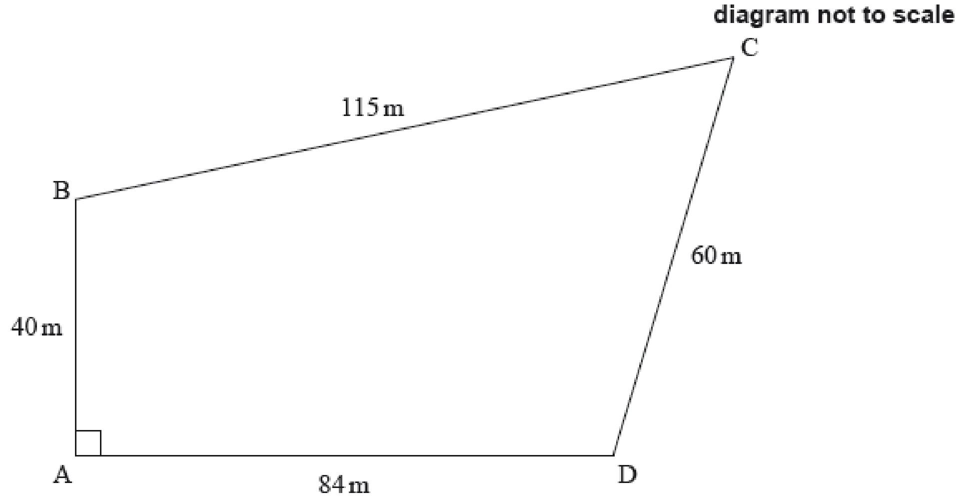
$$u_1 + u_2 + \ldots + u_n = 1512.$$

Find the value of n.

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD.

The lengths of the sides are $\mathrm{AB} = 40\ \mathrm{m}$, $\mathrm{BC} = 115\ \mathrm{m}$, $\mathrm{CD} = 60\ \mathrm{m}$, $\mathrm{AD} = 84\ \mathrm{m}$ and angle $\mathrm{B\hat{A}D} = 90^\circ$.

This information is shown on the diagram.



The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$$\text{area} = \frac{(\text{AB}+\text{CD})(\text{AD}+\text{BC})}{4}.$$

Abdallah uses this formula to estimate the area of his plot of land.

a. Show that $\text{BD} = 93\text{ m}$ correct to the nearest metre. [2]

b. Calculate angle $\text{B}\hat{\text{C}}\text{D}$. [3]

c. Find the area of ABCD. [4]

d.i. Calculate Abdallah's estimate for the area. [2]

d.ii. Find the percentage error in Abdallah's estimate. [2]

Give all your numerical answers correct to two decimal places.

On 1 January 2005, Daniel invested 30000 AUD at an annual **simple** interest rate in a *Regular Saver* account. On 1 January 2007, Daniel had 31650 AUD in the account.

b. On 1 January 2005, Rebecca invested 30000 AUD in a *Supersaver* account at a nominal annual rate of 2.5% **compounded annually**. [3]

Calculate the amount in the *Supersaver* account after two years.

c. On 1 January 2005, Rebecca invested 30000 AUD in a *Supersaver* account at a nominal annual rate of 2.5% **compounded annually**. [3]

Find the number of complete years since 1 January 2005 it would take for the amount in Rebecca's account to exceed the amount in Daniel's account.

d. On 1 January 2007, Daniel reinvested 80% of the money from the *Regular Saver* account in an *Extra Saver* account at a nominal annual rate of 3% **compounded quarterly**. [5]

(i) Calculate the amount of money reinvested by Daniel on the 1 January 2007.

(ii) Find the number of complete years it will take for the amount in Daniel's *Extra Saver* account to exceed 30000 AUD.

a. Prachi is on vacation in the United States. She is visiting the Grand Canyon. [2]

When she reaches the top, she drops a coin down a cliff. The coin falls down a distance of 5 metres during the first second, 15 metres during the next second, 25 metres during the third second and continues in this way. The distances that the coin falls during each second forms an arithmetic sequence.

(i) Write down the common difference, d , of this arithmetic sequence.

(ii) Write down the distance the coin falls during the fourth second.

b. Calculate the distance the coin falls during the 15th second. [2]

c. Calculate the **total** distance the coin falls in the first 15 seconds. Give your answer in kilometres. [3]

d. Prachi drops the coin from a height of 1800 metres above the ground. [3]

Calculate the time, to the nearest second, the coin will take to reach the ground.

e. Prachi visits a tourist centre nearby. It opened at the start of 2015 and in the first year there were $17\,000$ visitors. The number of people who visit the tourist centre is expected to increase by $10\,\%$ each year. [2]

Calculate the number of people expected to visit the tourist centre in 2016.

f. Calculate the total number of people expected to visit the tourist centre during the first 10 years since it opened. [3]

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

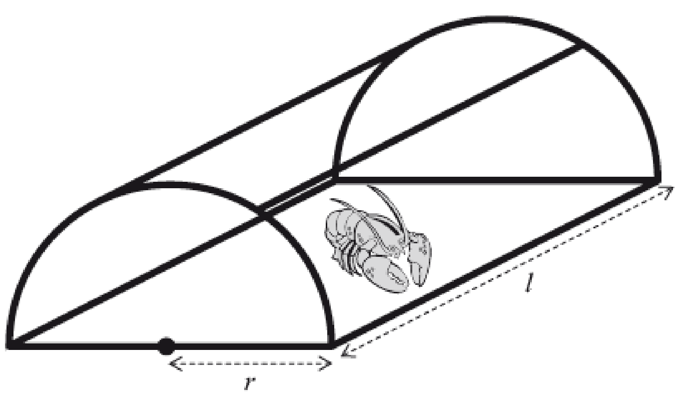


diagram not to scale

The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is 0.75 m^3. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is 0.75 m^3. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is 0.75 m^3. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is 0.719 m, correct to three significant figures.

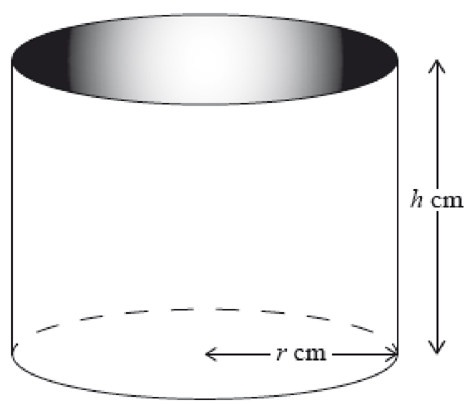
f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the value of l for which T is a minimum.

g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is 0.5 m^3.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of 2000 cm^2.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

On Monday Paco goes to a running track to train. He runs the first lap of the track in 120 seconds. Each lap Paco runs takes him 10 seconds longer than his previous lap.

a. Find the time, in seconds, Paco takes to run his fifth lap. [3]

b. Paco runs his last lap in 260 seconds. [3]

Find how many laps he has run on Monday.

c. Find the **total** time, in **minutes,** run by Paco on Monday. [4]

d. On Wednesday Paco takes Lola to train. They both run the first lap of the track in 120 seconds. Each lap Lola runs takes 1.06 times as long as her previous lap. [3]

Find the time, in seconds, Lola takes to run her third lap.

e. Find the **total** time, in seconds, Lola takes to run her first four laps. [3]

f. Each lap Paco runs again takes him 10 seconds longer than his previous lap. After a certain number of laps Paco takes less time per lap than Lola. [3]

Find the number of the lap when this happens.

Consider the function $f(x) = x^3 + \frac{48}{x}$, $x \neq 0$.

a. Calculate $f(2)$. [2]

b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. [4]

c. Find $f'(x)$. [3]

d. Find $f'(2)$. [2]

e. Write down the coordinates of the local maximum point on the graph of f. [2]

f. Find the range of f. [3]

g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]

h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

Find the x-coordinate of this point.

The Brahma chicken produces eggs with weights in grams that are normally distributed about a mean of 55 g with a standard deviation of 7 g. The eggs are classified as small, medium, large or extra large according to their weight, as shown in the table below.

Size	Weight (g)
Small	Weight < 53
Medium	$53 \le$ Weight < 63
Large	$63 \le$ Weight < 73
Extra Large	Weight ≥ 73

a. Sketch a diagram of the distribution of the weight of Brahma chicken eggs. On your diagram, show clearly the boundaries for the classification of the eggs. [3]

b. An egg is chosen at random. Find the probability that the egg is [4]

(i) medium;

(ii) extra large.

c. There is a probability of 0.3 that a randomly chosen egg weighs more than w grams. [2]

Find w.

d. The probability that a Brahma chicken produces a large size egg is 0.121. Frank's Brahma chickens produce 2000 eggs each month. [2]

Calculate an estimate of the number of large size eggs produced by Frank's chickens each month.

e. The selling price, in US dollars (USD), of each size is shown in the table below. [3]

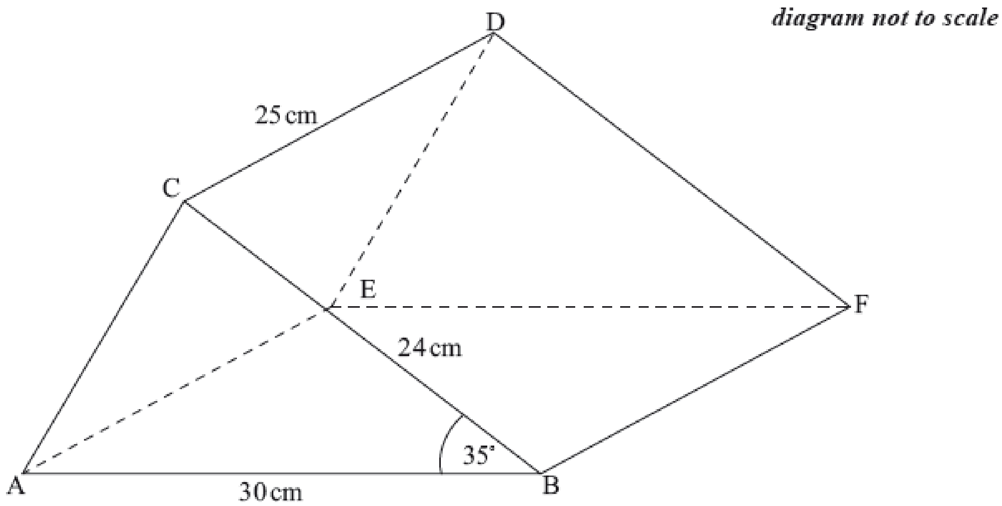
Size	Selling price (USD)
Small	0.30
Medium	0.50
Large	0.65
Extra Large	0.80

The probability that a Brahma chicken produces a small size egg is 0.388.

Estimate the monthly income, in USD, earned by selling the 2000 eggs. Give your answer correct to two decimal places.

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.

$\mathrm{AB} = 30\ \mathrm{cm},\ \mathrm{BC} = 24\ \mathrm{cm},\ \mathrm{CD} = 25\ \mathrm{cm}$ and angle $\mathrm{A\hat{B}C} = 35^\circ$.



a. Calculate the length of AC. [3]

b. Calculate the area of triangle ABC. [3]

c. Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\ \mathrm{cm}^3$, correct to three significant figures. [2]

d. Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Show that the total surface area of one block is $2190\ \mathrm{cm}^2$, correct to three significant figures. [3]

f. The blocks are to be painted. One litre of paint will cover $22\ \mathrm{m}^2$. [3]

Calculate the number of litres required to paint all 2600 blocks.

Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [3]

b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Show that $k = 3$.

c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f(2)$.

d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f'(2)$

e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]

Sketch the graph of $y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

State the values of x for which $f(x)$ is decreasing.

Cedric wants to buy an €8000 car. The car salesman offers him a loan repayment option of a 25 % deposit followed by 12 equal monthly payments of €600 .

a. Write down the amount of the deposit. [1]

b. Calculate the total cost of the loan under this repayment scheme. [2]

c. Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €*x* in the first month and €*y* in every following month until the €8000 is repaid. [1]

The total amount that Cedric's mother receives after **12** months is €3500. This can be written using the equation *x* +11*y* = 3500. The total amount that Cedric's mother receives after **24** months is €7100.

Write down a second equation involving *x* and *y*.

d. Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €*x* in the first month and €*y* in every following month until the €8000 is repaid. [2]

The total amount that Cedric's mother receives after **12** months is €3500. This can be written using the equation *x* +11*y* = 3500. The total amount that Cedric's mother receives after **24** months is €7100.

Write down the value of *x* and the value of *y*.

e. Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €*x* in the first month and €*y* in every following month until the €8000 is repaid. [3]

The total amount that Cedric's mother receives after **12** months is €3500. This can be written using the equation *x* +11*y* = 3500. The total amount that Cedric's mother receives after **24** months is €7100.

Calculate the number of months it will take Cedric's mother to receive the €8000.

f. Cedric decides to buy a cheaper car for €6000 and invests the remaining €2000 at his bank. The bank offers two investment options over three years. [5]

Option A: Compound interest at an annual rate of 8 %.

Option B: Compound interest at a nominal annual rate of 7.5 % , **compounded monthly**.

Express each answer in part (f) to the nearest euro.

Calculate the value of his investment at the end of three years if he chooses

(i) Option A;

(ii) Option B.

The following table shows the average body weight, x, and the average weight of the brain, y, of seven species of mammal. Both measured in kilograms (kg).

Species	Average body weight, x (kg)	Average weight of the brain, y (kg)
Cat	3	0.026
Cow	465	0.423
Donkey	187	0.419
Giraffe	529	0.680
Goat	28	0.115
Jaguar	100	0.157
Sheep	56	0.175

The average body weight of grey wolves is 36 kg.

In fact, the average weight of the brain of grey wolves is 0.120 kg.

The average body weight of mice is 0.023 kg.

a. Find the range of the average body weights for these seven species of mammal. [2]

b.i. For the data from these seven species calculate r, the Pearson’s product–moment correlation coefficient; [2]

b.ii. For the data from these seven species describe the correlation between the average body weight and the average weight of the brain. [2]

c. Write down the equation of the regression line y on x, in the form $y = mx + c$. [2]

d. Use your regression line to estimate the average weight of the brain of grey wolves. [2]

e. Find the percentage error in your estimate in part (d). [2]

f. State whether it is valid to use the regression line to estimate the average weight of the brain of mice. Give a reason for your answer. [2]

In a college 450 students were surveyed with the following results

150 have a television

205 have a computer

220 have an iPhone

75 have an iPhone and a computer

60 have a television and a computer

70 have a television and an iPhone

40 have all three.

a. Draw a Venn diagram to show this information. Use *T* to represent the set of students who have a television, *C* the set of students who have a computer and *I* the set of students who have an iPhone. [4]

b. Write down the number of students that [2]

(i) have a computer only;

(ii) have an iPhone and a computer but no television.

c. Write down $n[T \cap (C \cup I)']$. [1]

d. Calculate the number of students who have none of the three. [2]

e. Two students are chosen at random from the 450 students. Calculate the probability that [6]

(i) neither student has an iPhone;

(ii) only one of the students has an iPhone.

f. The students are asked to collect money for charity. In the first month, the students collect *x* dollars and the students collect *y* dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$. [3]

In the first 10 months they collect 13 050 dollars.

(i) Write down a second equation in *x* and *y* to represent this information.

(ii) Write down the value of *x* and of *y* .

g. The students are asked to collect money for charity. In the first month, the students collect *x* dollars and the students collect *y* dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$. [3]

In the first 10 months they collect 13 050 dollars.

Calculate the number of months that it will take the students to collect 49 500 dollars.

Consider the function $f(x) = x^3 - 3\text{x} - 24x + 30$.

a. Write down *f* (0). [1]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. [2]

d. (i) Use $f'(x)$ to find the x-coordinate of M and of N. [5]

(ii) Hence or otherwise write down the coordinates of M and of N.

e. Sketch the graph of $f(x)$ for $-5 \leqslant x \leqslant 7$ and $-60 \leqslant y \leqslant 60$. Mark clearly M and N on your graph. [4]

f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. [6]

(i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

Part A

100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X) , bread (Y) and fruit (Z). It is found that

10 students had all three

17 students had bread and fruit only

15 students had cereal and fruit only

12 students had cereal and bread only

13 students had only bread

8 students had only cereal

9 students had only fruit

Part B

The same 100 students are also asked how many meals on average they have per day. The data collected is organized in the following table.

	3 or fewer meals per day	4 or 5 meals per day	More than 5 meals per day	Total
Male	15	25	15	55
Female	12	20	13	45
Total	27	45	28	100

A χ^2 test is carried out at the 5 % level of significance.

A.a Represent this information on a Venn diagram. [4]

A.b Find the number of students who had none of the three choices for breakfast. [2]

A.c Write down the percentage of students who had fruit for breakfast. [2]

A.d. Describe in words what the students in the set $X \cap Y'$ had for breakfast. [2]

A.e. Find the probability that a student had **at least** two of the three choices for breakfast. [2]

A.f. Two students are chosen at random. Find the probability that both students had all three choices for breakfast. [3]

B.a. Write down the null hypothesis, H_0, for this test. [1]

B.b. Write down the number of degrees of freedom for this test. [1]

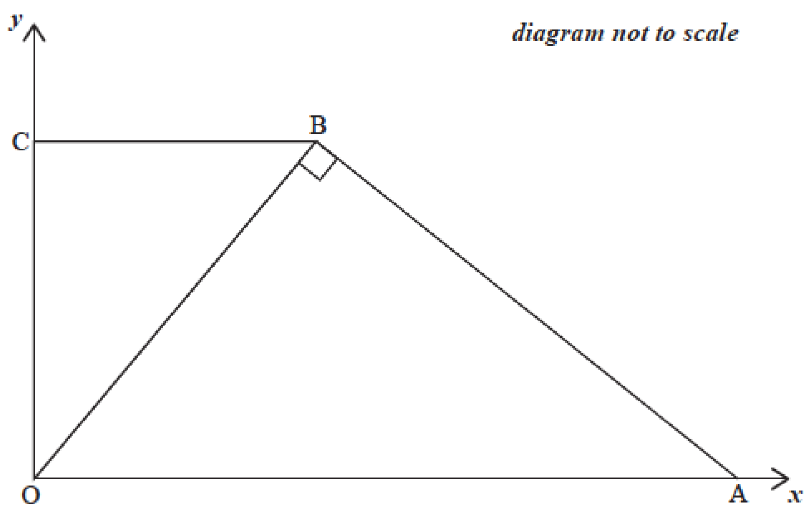
B.c. Write down the critical value for this test. [1]

B.d. Show that the expected number of females that have more than 5 meals per day is 13, correct to the nearest integer. [2]

B.e. Use your graphic display calculator to find the χ^2_{calc} for this data. [2]

B.f. Decide whether H_0 must be accepted. Justify your answer. [2]

The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y-axis, and O is the origin.



a. The equation of the line BC is $y = 4$. [1]

Write down the coordinates of point C.

b. The x-coordinate of point B is a. [2]

(i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB.

c. Point A lies on the x-axis and the line AB is perpendicular to line OB. [4]

(i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$.

d. The area of triangle AOB is **three times** the area of triangle OBC. [3]

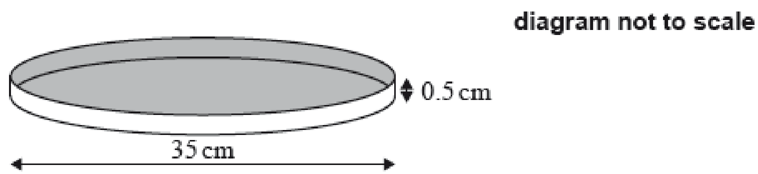
Find an expression, **in terms of *a***, for

(i) the area of triangle OBC;

(ii) the x-coordinate of point A.

e. Calculate the value of a. [2]

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.



A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, °C, can be modelled by

$$P(t) = a(2.06)^{-t} + 19,\ t \geqslant 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was 230 °C.

The pizza can be eaten once its temperature drops to 45 °C.

a. Calculate the volume of this pan. [3]

b. Find the radius of the sphere in cm, correct to one decimal place. [4]

c. Find the value of a. [2]

d. Find the temperature that the pizza will be 5 minutes after it is taken out of the oven. [2]

e. Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten. [3]

f. In the context of this model, state what the value of 19 represents. [1]

---

a. Antonio and Barbara start work at the same company on the same day. They each earn an annual salary of 8000 euros during the first year of employment. The company gives them a salary increase following the completion of each year of employment. Antonio is paid using plan A and Barbara is paid using plan B. [3]

Plan A: The annual salary increases by 450 euros each year.

Plan B: The annual salary increases by $5\ \%$ each year.

Calculate

i) Antonio’s annual salary during his second year of employment;

ii) Barbara’s annual salary during her second year of employment.

b. Write down an expression for [4]

i) Antonio’s annual salary during his n th year of employment;

ii) Barbara’s annual salary during her n th year of employment.

c. Determine the number of years for which Antonio’s annual salary is greater than or equal to Barbara’s annual salary. [2]

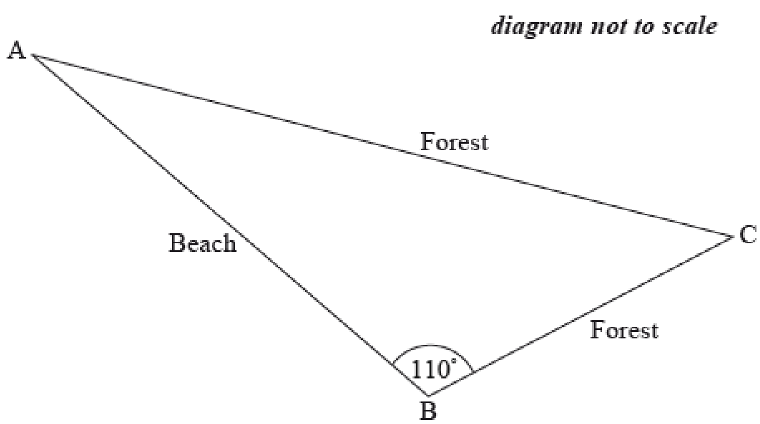
d. Both Antonio and Barbara plan to work at the company for a total of 15 years. [7]

i) Calculate the **total amount** that **Barbara** will be paid during these 15 years.

ii) Determine whether Antonio earns more than Barbara during these 15 years.

A cross-country running course consists of a beach section and a forest section. Competitors run from A to B, then from B to C and from C back to A.

The running course from A to B is along the beach, while the course from B, through C and back to A, is through the forest.
The course is shown on the following diagram.



Angle ABC is $110°$.
It takes Sarah 5 minutes and 20 seconds to run from A to B at a speed of $3.8\ \mathrm{ms}^{-1}$.

a. Using ‘*distance* = *speed* $\times$ *time*’, show that the distance from A to B is 1220 metres correct to 3 significant figures. [2]

b. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [1]

Calculate the speed, in ms^{-1}, that Sarah runs from B to C.

c. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the distance, in metres, from $\mathbf{C}$ **to** $\mathbf{A}$.

d. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [2]

Calculate the total distance, in metres, of the cross-country running course.

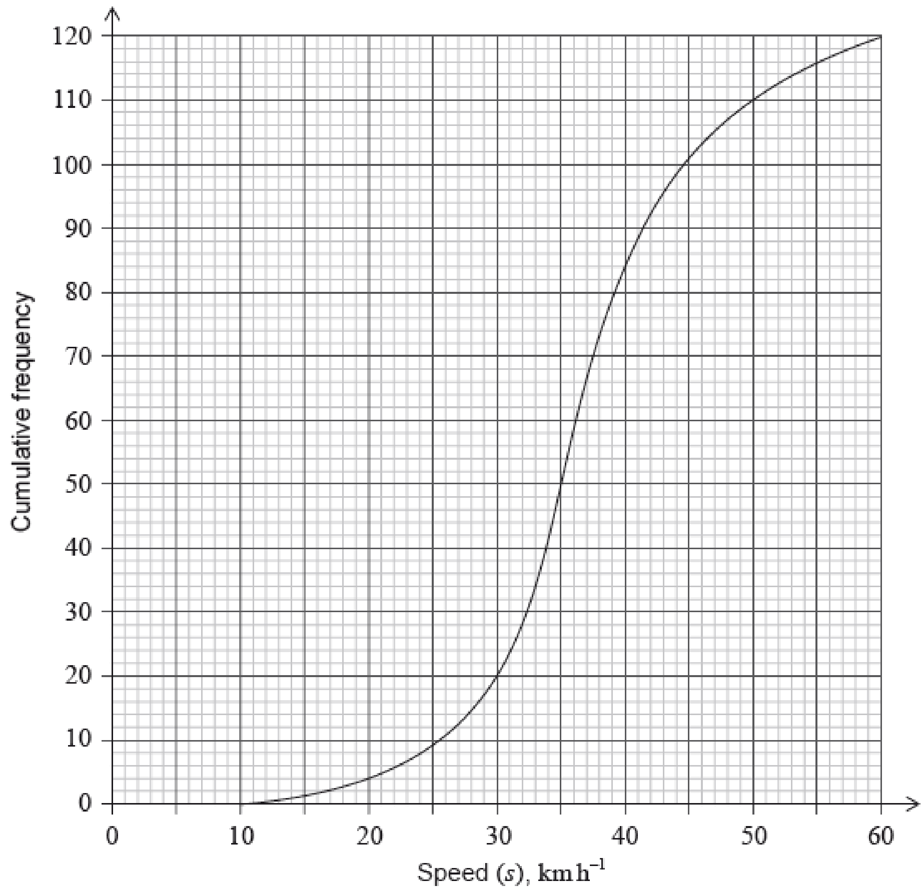
e. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Find the size of angle BCA.

f. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the area of the cross-country course bounded by the lines AB, BC and CA.

The cumulative frequency graph shows the speed, s, in $\mathrm{km\,h^{-1}}$, of 120 vehicles passing a hospital gate.



a. Estimate the minimum possible speed of one of these vehicles passing the hospital gate. [1]

b. Find the median speed of the vehicles. [2]

c. Write down the 75^{th} percentile. [1]

d. Calculate the interquartile range. [2]

e. The speed limit past the hospital gate is $50\ \mathrm{km\,h^{-1}}$. [2]

Find the number of these vehicles that exceed the speed limit.

f. The table shows the speeds of these vehicles travelling past the hospital gate. [2]

Speed of Vehicles	Number of Vehicles
$0 < s \leq 10$	0
$10 < s \leq 20$	p
$20 < s \leq 30$	16
$30 < s \leq 40$	64
$40 < s \leq 50$	26
$50 < s \leq 60$	q

Find the value of p and of q.

g. The table shows the speeds of these vehicles travelling past the hospital gate. [2]

Speed of Vehicles	Number of Vehicles
$0 < s \leq 10$	0
$10 < s \leq 20$	p
$20 < s \leq 30$	16
$30 < s \leq 40$	64
$40 < s \leq 50$	26
$50 < s \leq 60$	q

(i) Write down the modal class.

(ii) Write down the mid-interval value for this class.

h. The table shows the speeds of these vehicles travelling past the hospital gate. [3]

Speed of Vehicles	Number of Vehicles
$0 < s \leq 10$	0
$10 < s \leq 20$	p
$20 < s \leq 30$	16
$30 < s \leq 40$	64
$40 < s \leq 50$	26
$50 < s \leq 60$	q

Use your graphic display calculator to calculate an estimate of

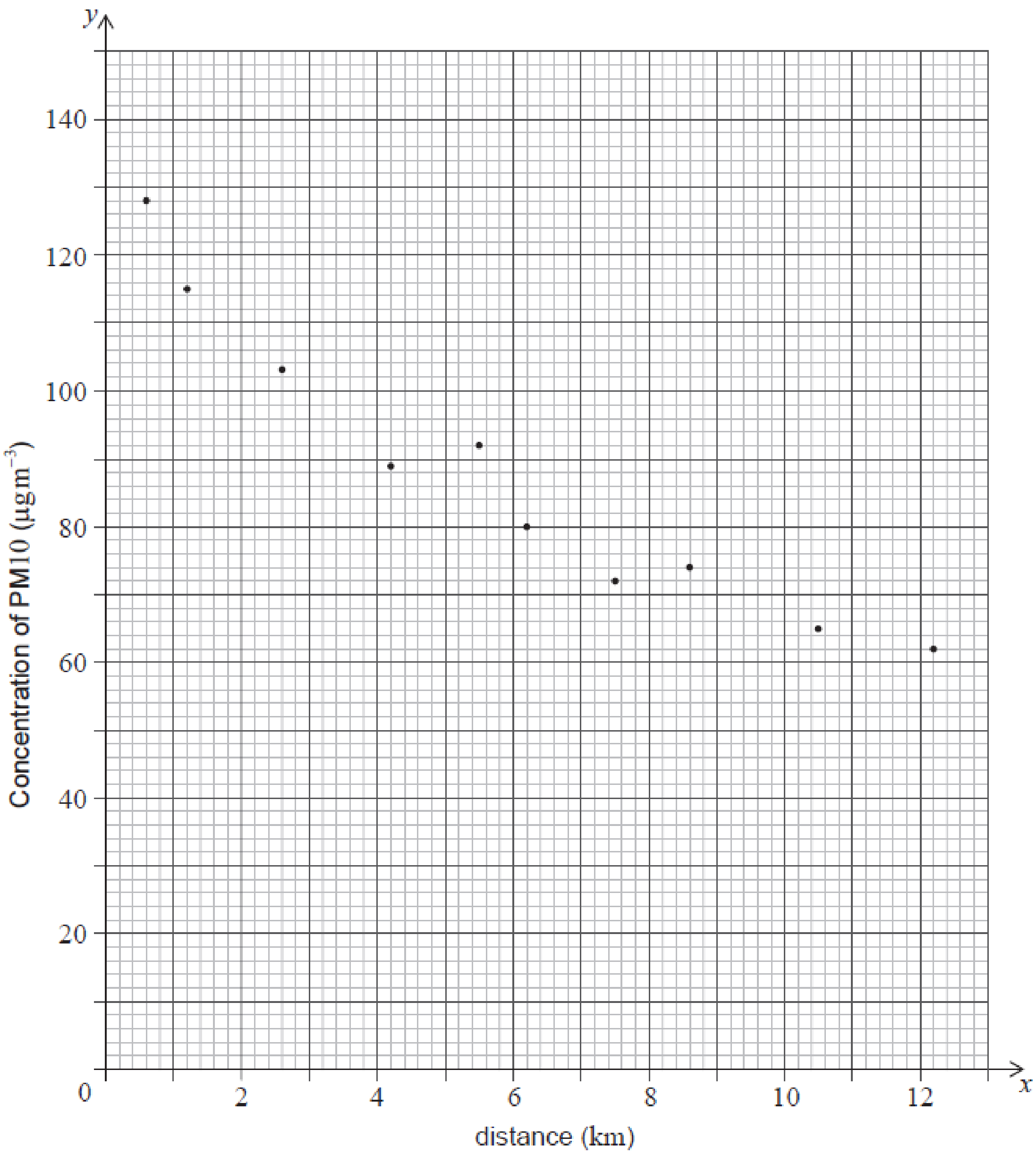
(i) the mean speed of these vehicles;

(ii) the standard deviation.

i. It is proposed that the speed limit past the hospital gate is reduced to $40\ \mathrm{km\,h^{-1}}$ from the current $50\ \mathrm{km\,h^{-1}}$. [2]

Find the percentage of these vehicles passing the hospital gate that **do not** exceed the current speed limit but **would** exceed the new speed limit.

a. For an ecological study, Ernesto measured the average concentration (y) of the fine dust, $\mathrm{PM10}$, in the air at different distances (x) from a power plant. His data are represented on the following scatter diagram. The concentration of $\mathrm{PM10}$ is measured in micrograms per cubic metre and the distance is measured in kilometres. [2]



His data are also listed in the following table.

Distance (x)	0.6	1.2	2.6	a	5.5	6.2	7.5	8.6	10.5	12.2
Concentration of PM10 (y)	128	115	103	89	92	80	72	b	65	62

Use the scatter diagram to find the value of a and of b in the table.

b. Calculate [4]

i) $\bar{x}$, the mean distance from the power plant;

ii) $\bar{y}$, the mean concentration of $\mathrm{PM10}$;

iii) r , the Pearson's product–moment correlation coefficient.

c. Write down the equation of the regression line y on x . [2]

d. Ernesto's school is located $14\,\text{km}$ from the power plant. He uses the equation of the regression line to estimate the concentration of PM10 in the air at his school. [4]

i) Calculate the value of Ernesto's estimate.

ii) State whether Ernesto's estimate is reliable. Justify your answer.

Throughout this question *all* the numerical answers must be given correct to the nearest whole number.

a. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. [4]

Find the number of students attending Park School in

(i) January 2001;

(ii) January 2003.

b. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. [2]

Show that the number of students attending Park School in January 2007 is 150.

c. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. [2]

Find the number of students attending Grove School in January 2003.

d. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. [4]

Find the year in which the number of students attending Grove School will be first 60% **more than** in January 2000.

e. Each January, one of these two schools, the one that has more students, is given extra money to spend on sports equipment. [5]

(i) Decide which school gets the money in 2007. Justify your answer.

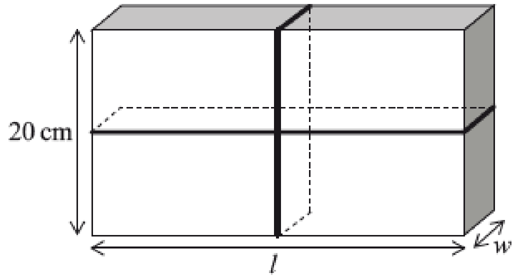
(ii) Find the first year in which Park School will be given this extra money.

A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm. The total volume of the parcel is $3000\ \text{cm}^3$.

a. Express the volume of the parcel in terms of l and w. [1]

b. Show that $l = \frac{150}{w}$. [2]

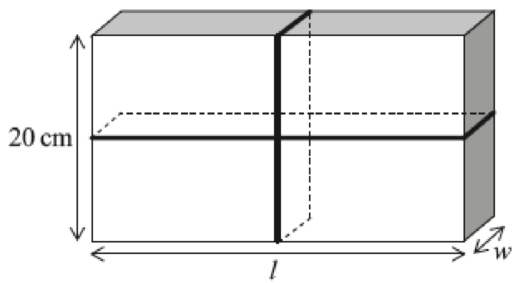
c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

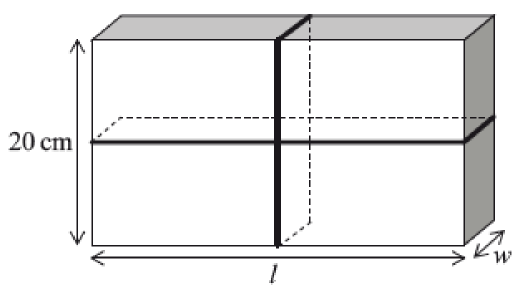
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



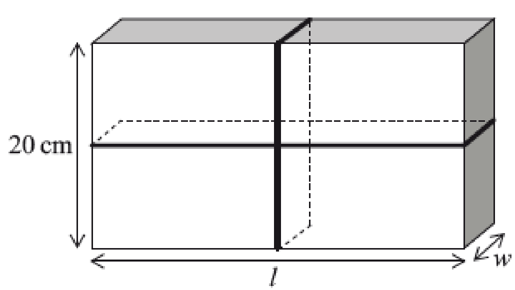
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



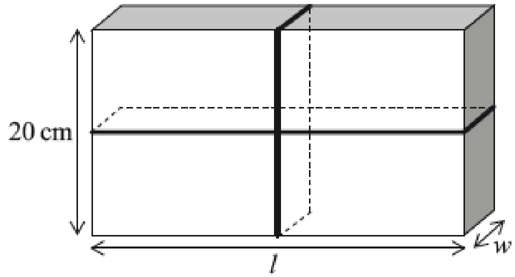
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



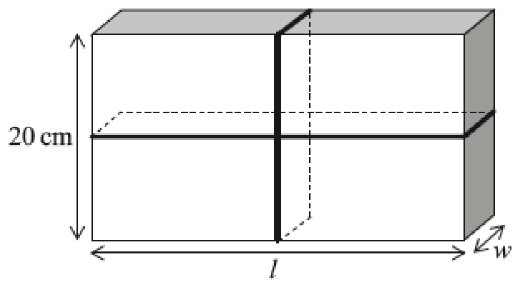
Find the value of w for which S is a minimum.

g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [1]



Write down the value, l, of the parcel for which the length of string is a minimum.

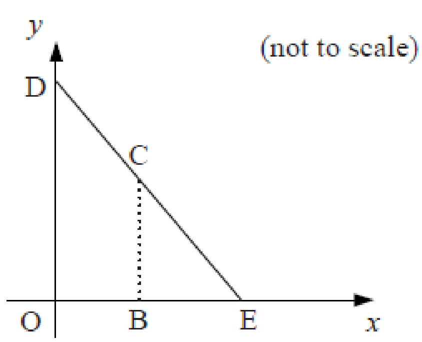
h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Find the minimum length of string required to tie up the parcel.

On the coordinate axes below, D is a point on the y-axis and E is a point on the x-axis. O is the origin. The equation of the line DE is $y+\frac{1}{2}x=4$

.



a. Write down the coordinates of point E. [2]

b. C is a point on the line DE. B is a point on the x-axis such that BC is parallel to the y-axis. The x-coordinate of C is t. [2]

Show that the y-coordinate of C is $4-\frac{1}{2}t$.

c. OBCD is a trapezium. The y-coordinate of point D is 4. [3]

Show that the area of OBCD is $4t-\frac{1}{4}t^2$.

d. The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information. [1]

e. (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d). [4]

(ii) Hence find the correct value for t. Give a reason for your answer.

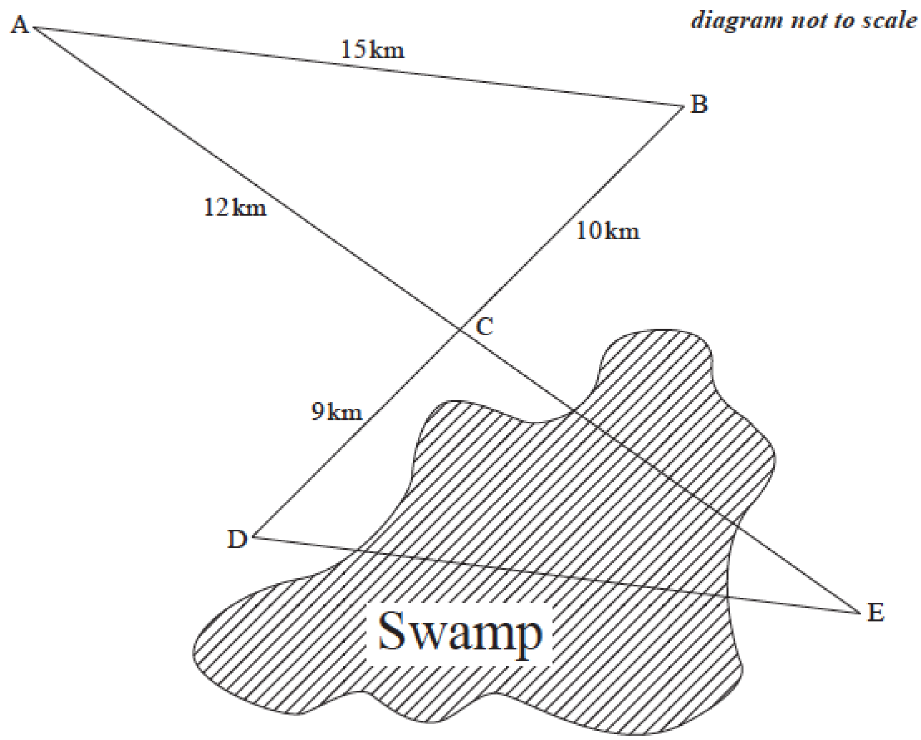
A surveyor has to calculate the area of a triangular piece of land, DCE.

The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.

The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor's information.



a. (i) Find the size of angle ACB. [4]

(ii) Show that the size of angle DCE is 85.5°, correct to one decimal place.

b. The surveyor measures the size of angle CDE to be twice that of angle DEC. [5]

(i) Using angle $\mathrm{DCE} = 85.5^{\circ}$, find the size of angle DEC.

(ii) Find the length of DE.

c. Calculate the area of triangle DEC. [4]

The following table shows the number of bicycles, x, produced daily by a factory and their total production cost, y, in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of bicycles, x	12	15	14	17	20	18	21
Production cost, y	3900	4600	4100	5300	6000	5400	6000

a. (i) Write down the Pearson's product–moment correlation coefficient, r, for these data. [4]

(ii) Hence comment on the result.

b. Write down the equation of the regression line y on x for these data, in the form $y = ax + b$. [2]

c. Estimate the total cost, **to the nearest USD**, of producing 13 bicycles on a particular day. [3]

d. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. [2]

Explain why the factory does **not** make a profit when producing 13 bicycles on a particular day.

e. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. [5]

(i) Write down an expression for the total selling price of x bicycles.

(ii) Write down an expression for the **profit** the factory makes when producing x bicycles on a particular day.

(iii) Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit.

Give all answers in this question correct to two decimal places.

Arthur lives in London. On 1^{st} August 2008 Arthur paid $37\,500$ euros (EUR) for a new car from Germany. The price of the same car in London was $34\,075$ British pounds (GBP).
The exchange rate on 1^{st} August 2008 was $1\,\text{EUR} = 0.7234\,\text{GBP}$.

a. Calculate, **in GBP**, the price that Arthur paid for the car. [2]

b. Write down, in GBP, the amount of money Arthur saved by buying the car in Germany. [1]

d. Between 1^{st} August 2008 and 1^{st} August 2012 Arthur's car depreciated at an annual rate of 9% of its current value. [3]

Calculate the value, in GBP, of Arthur's car on 1^{st} August **2009**.

e. Between 1^{st} August 2008 and 1^{st} August 2012 Arthur's car depreciated at an annual rate of 9% of its current value. [3]

Show that the value of Arthur's car on 1^{st} August **2012** was $18\,600\,\text{GBP}$, correct to the nearest $100\,\text{GBP}$.

Consider the functions $f(x) = \frac{2x+3}{x+4}$ and $g(x) = x + 0.5$.

a. Sketch the graph of the function $f(x)$, for $-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. [6]

b. Write down the equation of the vertical asymptote. [2]

c. On the same diagram as part (a) sketch the graph of $g(x) = x + 0.5$. [2]

d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**. [3]

e. Write down the gradient of the line $g(x) = x + 0.5$. [1]

f. The line L passes through the point with coordinates $(-2, -3)$ and is perpendicular to the line $g(x)$. Find the equation of L. [3]

The table shows the distance, in km, of eight regional railway stations from a city centre terminus and the price, in \$, of a return ticket from each regional station to the terminus.

Distance in km (x)	3	15	23	42	56	62	74	93
Price in \$ (y)	5	24	43	56	68	74	86	100

a. Draw a scatter diagram for the above data. Use a scale of 1 cm to represent 10 km on the x-axis and 1 cm to represent $\$10$ on the y-axis. [4]

b. Use your graphic display calculator to find [2]

(i) $\bar{x}$, the mean of the distances;

(ii) $\bar{y}$, the mean of the prices.

c. Plot and label the point $\mathrm{M}\ (\bar{x},\ \bar{y})$ on your scatter diagram. [1]

d. Use your graphic display calculator to find [3]

(i) the product–moment correlation coefficient, r ;

(ii) the equation of the regression line y on x.

e. Draw the regression line y on x on your scatter diagram. [2]

f. A ninth regional station is 76 km from the city centre terminus. [3]

Use the equation of the regression line to estimate the price of a return ticket to the city centre terminus from this regional station. **Give your answer correct to the nearest \$**.

g. Give a reason why it is valid to use your regression line to estimate the price of this return ticket. [1]

h. The actual price of the return ticket is $\$80$. [2]

Using your answer to part (f), calculate the percentage error in the estimated price of the ticket.

Give your answers to parts (b), (c) and (d) to the nearest whole number.

Harinder has 14 000 US Dollars (USD) to invest for a period of five years. He has two options of how to invest the money.

Option A: Invest the full amount, in USD, in a fixed deposit account in an American bank.

The account pays a nominal annual interest rate of $r\%$, **compounded yearly**, for the five years. The bank manager says that this will give Harinder a return of 17500 USD.

Option B: Invest the full amount, in Indian Rupees (INR), in a fixed deposit account in an Indian bank. The money must be converted from USD to INR before it is invested.

The exchange rate is 1 USD = 66.91 INR.

The account in the Indian bank pays a nominal annual interest rate of 5.2 % **compounded monthly**.

a. Calculate the value of *r*. [3]

b. Calculate 14 000 USD in INR. [2]

c. Calculate the amount of this investment, in INR, in this account after five years. [3]

d. Harinder chose option B. At the end of five years, Harinder converted this investment back to USD. The exchange rate, at that time, was 1 USD = 67.16 INR. [3]

Calculate how much **more** money, in USD, Harinder earned by choosing option B instead of option A.

Daniel grows apples and chooses at random a sample of 100 apples from his harvest.

He measures the diameters of the apples to the nearest cm. The following table shows the distribution of the diameters.

Diameter (to the nearest cm)	5	6	7	8	9
Frequency	15	27	33	17	8

a. Using your graphic display calculator, write down the value of [3]

(i) the mean of the diameters in this sample;

(ii) the standard deviation of the diameters in this sample.

b. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [3]

Classification	**Diameter (cm)**
Small	Diameter < 6.5
Medium	$6.5 \leq$ Diameter $< a$
Large	Diameter $\geq a$

Calculate the percentage of **small** apples in Daniel's harvest.

c. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [2]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \le$ Diameter $< a$
Large	Diameter $\ge a$

Of the apples harvested, 5% are **large** apples.

Find the value of a.

d. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [2]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \le$ Diameter $< a$
Large	Diameter $\ge a$

Find the percentage of **medium** apples.

e. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [2]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \le$ Diameter $< a$
Large	Diameter $\ge a$

This year, Daniel estimates that he will grow $100\,000$ apples.

Estimate the number of **large** apples that Daniel will grow this year.

A boat race takes place around a triangular course, ABC, with $\mathrm{AB} = 700\ \mathrm{m}$, $\mathrm{BC} = 900\ \mathrm{m}$ and angle $\mathrm{ABC} = 110^{\circ}$. The race starts and finishes at point A.

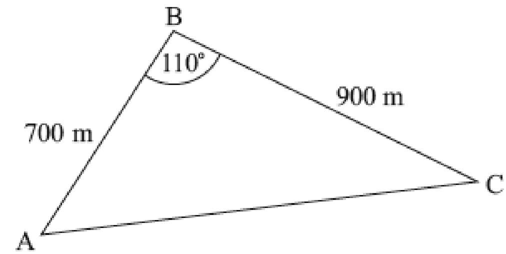


diagram not to scale

a. Calculate the total length of the course. [4]

b. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Calculate an estimate of the winning time of the race. Give your answer to the nearest minute.

c. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Find the size of angle ACB.

d. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Calculate the area that must be kept clear of boats.

e. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Determine, giving a reason, whether the course complies with the safety regulations.

f. The race is filmed from a helicopter, H, which is flying vertically above point A. [2]

The angle of elevation of H from B is 15°.

Calculate the vertical height, AH, of the helicopter above A.

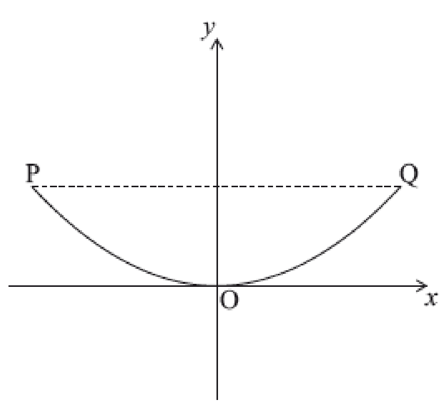
g. The race is filmed from a helicopter, H, which is flying vertically above point A. [3]

The angle of elevation of H from B is 15°.

Calculate the maximum possible distance from the helicopter to a boat on the course.

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



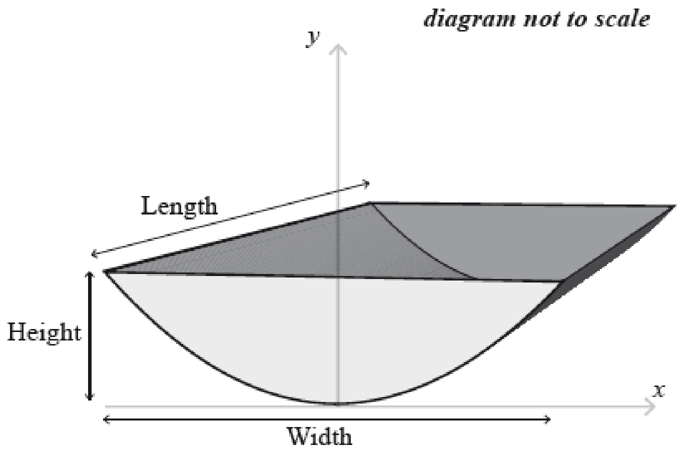
Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

a. Write down the value of c. [1]

b. Find the value of a. [2]

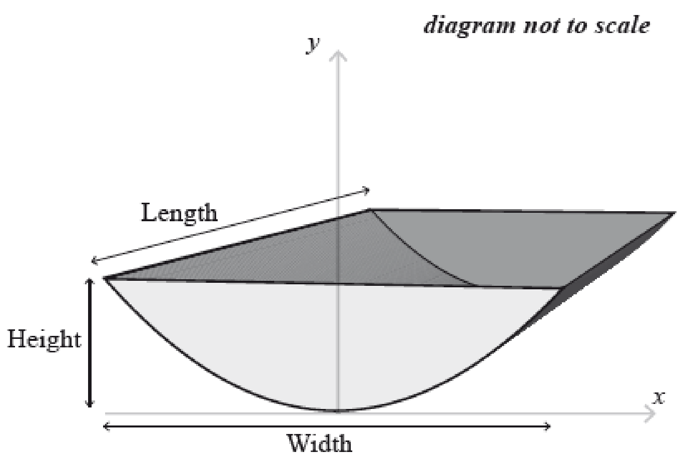
c. Hence write down the equation of the quadratic function which models the edge of the water tank. [1]

d. The water tank is shown below. It is partially filled with water. [2]



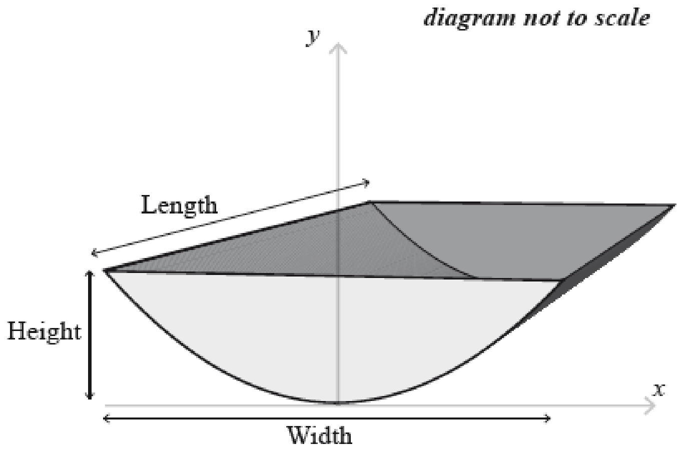
Calculate the value of y when $x = 2.4\text{ m}$.

e. The water tank is shown below. It is partially filled with water. [2]



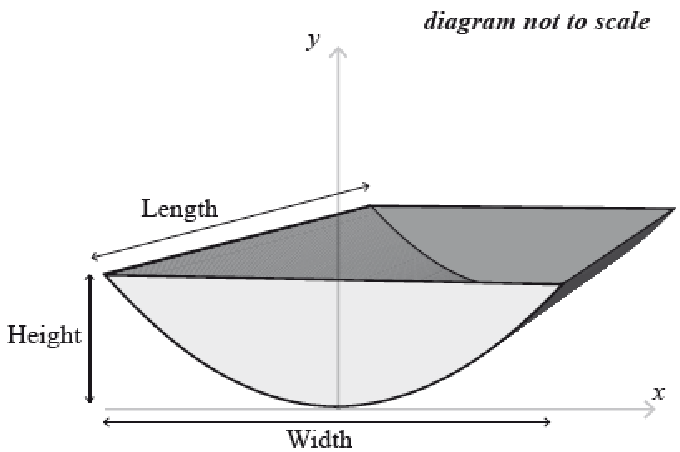
State what the value of x and the value of y represent for this water tank.

f. The water tank is shown below. It is partially filled with water. [2]



Find the value of x when the height of water in the tank is 0.9 m.

g. The water tank is shown below. It is partially filled with water. [2]



The water tank has a length of 5 m.

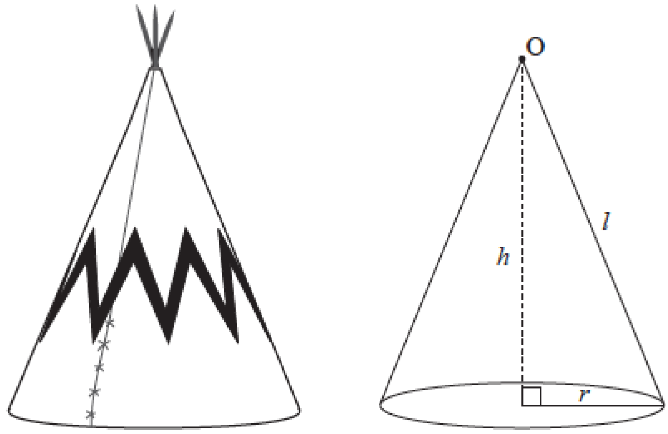
When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is $2.55\ \mathrm{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ \mathrm{m}^3$.

(ii) Calculate the percentage of water in the tank.

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is $39.27\ \mathrm{m}^2$, and has the shape of a semicircle, as shown in the following diagram.

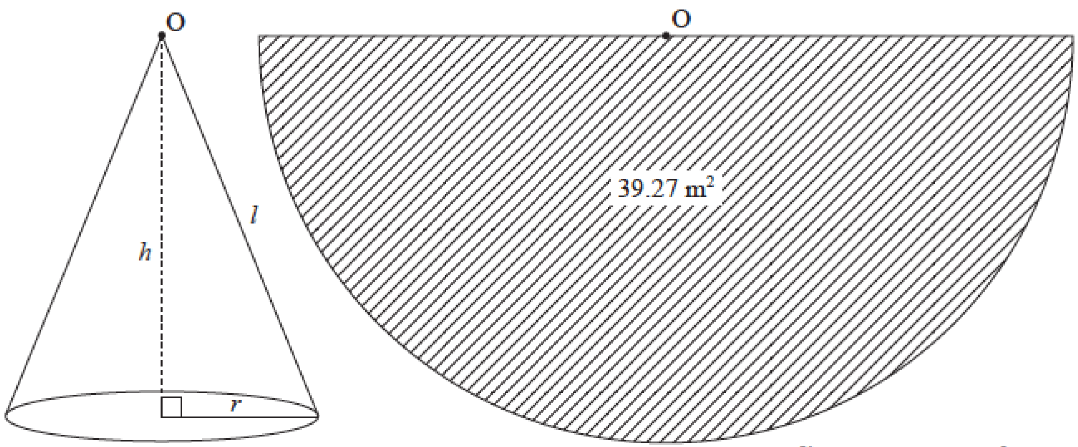


diagram not to scale

a. Show that the slant height, l, is 5 m, correct to the nearest metre. [2]

b. (i) Find the circumference of the base of the cone. [6]

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

c. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

d. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

e. A company designs cone-shaped tents to resemble the traditional tepees. [2]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

f. A company designs cone-shaped tents to resemble the traditional tepees. [4]

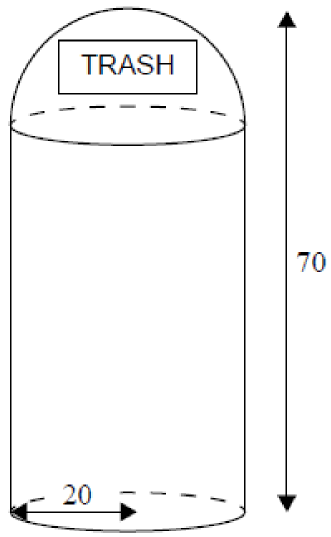
These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

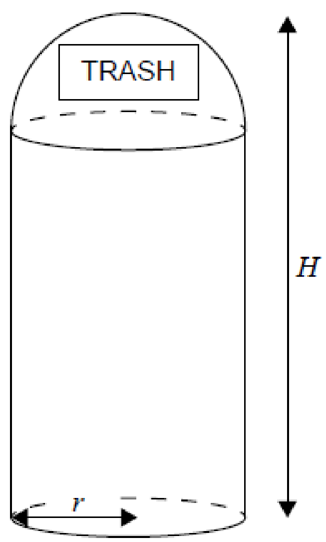


A designer is asked to produce a new trash can.

The new trash can will also be in the form of a cylinder with a hemispherical top.

This trash can will have a height of H cm and a base radius of r cm.

diagram not to scale



There is a design constraint such that $H + 2r = 110$ cm.

The designer has to maximize the volume of the trash can.

a. Write down the height of the cylinder. [1]

b. Find the total volume of the trash can. [4]

c. Find the height of the **cylinder**, h , of the new trash can, in terms of r. [2]

d. Show that the volume, V cm^3 , of the new trash can is given by [3]

$V = 110\pi r^3$.

e. Using your graphic display calculator, find the value of r which maximizes the value of V. [2]

f. The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can. [4]

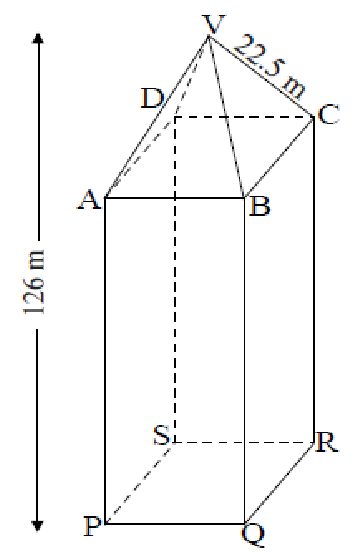
State whether the designer’s claim is correct. Justify your answer.

The diagram shows an office tower of total height 126 metres. It consists of a square based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1°.

diagram not to scale



a.i. Write down the length of VA in metres. [1]

a.ii. Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. [1]

b. Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. [2]

c. Calculate the length of AC in metres. [3]

d. Show that the length of BC is 19.1 metres correct to 3 significant figures. [2]

e. Calculate the volume of the tower. [4]

f. To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg. [3]

Calculate the weight of air in the tower.

A, a. Sketch the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. Indicate clearly where the curve intersects the y-axis. [3]

A, b. Write down the equation of the asymptote of the graph of $y = 2^x$. [2]

A, c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3]

A, d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2]

A, e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. [1]

A, f. Use Differential Calculus to verify that your answer to (e) is correct. [5]

B, a. The curve $y = px^2 + qx - 4$ passes through the point (2, –10). [2]

Use the above information to write down an equation in p and q.

B, b. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [2]

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

B, b. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [1]

Hence, find a second equation in p and q.

B, c. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [3]

Solve the equations to find the value of p and of q.

A shipping container is to be made with six rectangular faces, as shown in the diagram.

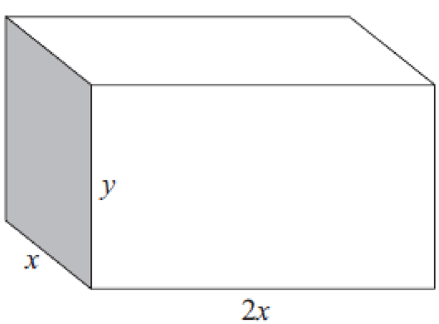


diagram not to scale

The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

a. Show that $y = 12 - 3x$. [3]

b. Show that the volume V m^3 of the container is given by [2]

$V = 24x^2 - 6x^3$

c. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

d. Find the value of x for which V is a maximum. [3]

e. Find the maximum volume of the container. [2]

f. Find the length and height of the container for which the volume is a maximum. [3]

g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. [4]

Calculate the number of tins required to paint the shipping container.

Beartown has three local newspapers: *The Art Journal*, *The Beartown News*, and *The Currier*.

A survey shows that

32 % of the town's population read *The Art Journal*,

46 % read *The Beartown News*,

54 % read *The Currier*,

3 % read *The Art Journal* and *The Beartown News* **only**,

8 % read *The Art Journal* and *The Currier* **only**,

12 % read *The Beartown News* and *The Currier* **only**, and

5 % of the population reads **all** three newspapers.

a. Draw a Venn diagram to represent this information. Label A the set that represents *The Art Journal* readers, B the set that represents *The Beartown News* readers, and C the set that represents *The Currier* readers. [4]

b. Find the percentage of the population that does not read any of the three newspapers. [2]

c. Find the percentage of the population that reads exactly one newspaper. [2]

d. Find the percentage of the population that reads *The Art Journal* or *The Beartown News* but not *The Currier*. [2]

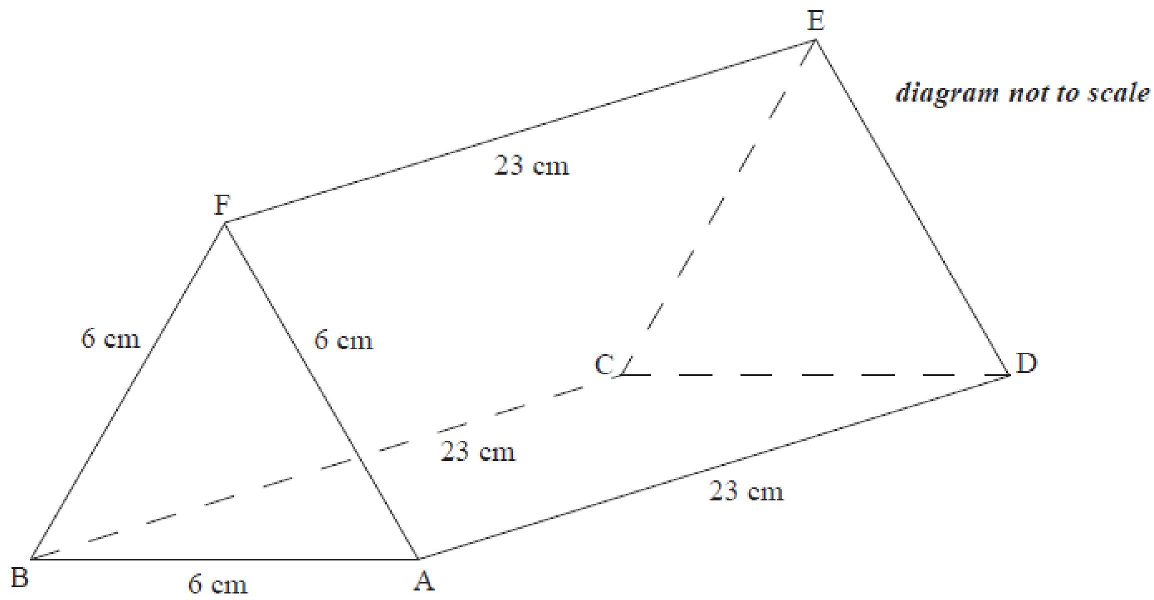
e. A local radio station states that 83 % of the population reads either *The Beartown News* or *The Currier*. [2]

Use your Venn diagram to decide whether the statement is true. Justify your answer.

f. The population of Beartown is 120 000. The local radio station claimed that 34 000 of the town's citizens read at least two of the local newspapers. [4]

Find the percentage error in this claim.

A chocolate bar has the shape of a triangular right prism ABCDEF as shown in the diagram. The ends are equilateral triangles of side 6 cm and the length of the chocolate bar is 23 cm.



a, i. Write down the size of angle BAF. [1]

a, ii. Hence or otherwise find the area of the triangular end of the chocolate bar. [3]

b. Find the total surface area of the chocolate bar. [3]

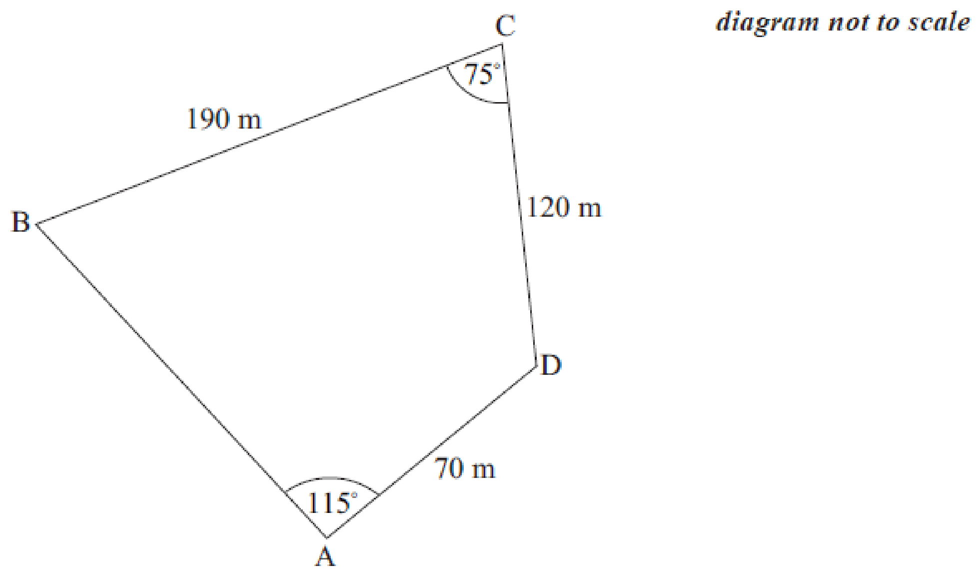
c. It is known that 1 cm^3 of this chocolate weighs 1.5 g. Calculate the weight of the chocolate bar. [3]

d. A different chocolate bar made with the same mixture also has the shape of a triangular prism. The ends are triangles with sides of length 4 cm, 6 cm and 7 cm. [3]

Show that the size of the angle between the sides of 6 cm and 4 cm is 86.4° correct to 3 significant figures.

e. The weight of this chocolate bar is 500 g. Find its length. [4]

Pauline owns a piece of land ABCD in the shape of a quadrilateral. The length of BC is 190 m , the length of CD is 120 m , the length of AD is 70 m , the size of angle BCD is 75° and the size of angle BAD is 115° .



Pauline decides to sell the triangular portion of land ABD . She first builds a straight fence from B to D .

a. Calculate the length of the fence. [3]

b. The fence costs 17 USD per metre to build. [2]

Calculate the cost of building the fence. Give your answer correct to the nearest USD.

c. Show that the size of angle ABD is 18.8° , correct to three significant figures. [3]

d. Calculate the area of triangle ABD . [4]

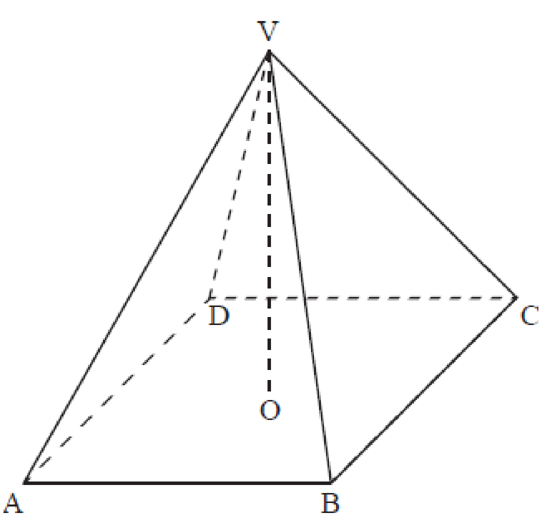
e. She sells the land for 120 USD per square metre. [2]

Calculate the value of the land that Pauline sells. Give your answer correct to the nearest USD.

f. Pauline invests 300000 USD from the sale of the land in a bank that pays compound interest compounded annually. [4]

Find the interest rate that the bank pays so that the investment will double in value in 15 years.

ABCDV is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.



a. Calculate the volume of the pyramid. [2]

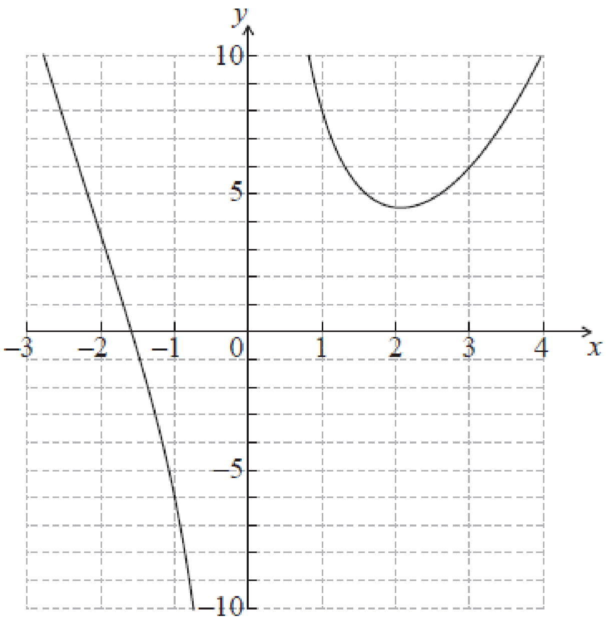
b. The glass weighs 9.3 grams per cm^3. Calculate the weight of the pyramid. [2]

c. Show that the length of the sloping edge VC of the pyramid is 3.6 cm. [4]

d. Calculate the angle at the vertex, $\mathrm{B\hat{V}C}$. [3]

e. Calculate the total surface area of the pyramid. [4]

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down [5]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

b. Find $f'(x)$. [4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$. [2]

d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. [2]

e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A. [3]

Find the equation of L . Give your answer in the form $y = mx + c$.

g. The line, L , passes through the point A and is perpendicular to the tangent at A. [2]

L also intersects the graph of $y = f(x)$ at points B and C . Write down the ***x*-coordinate** of B and of C .

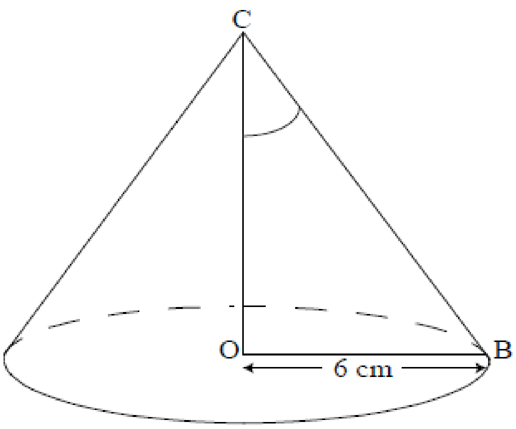
A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

a. Find the area of the base of the cylinder. [2]

b. Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures. [2]

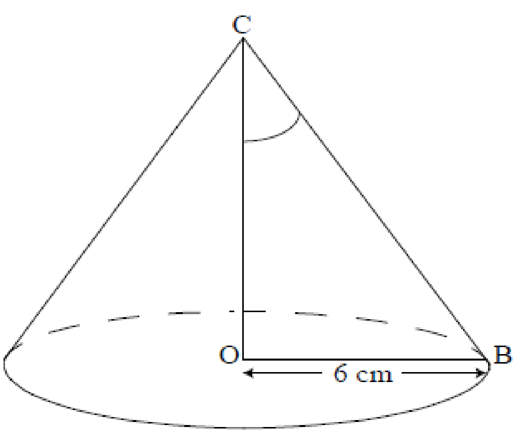
c. Find the total surface area of the cylinder. [3]

d. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [3]



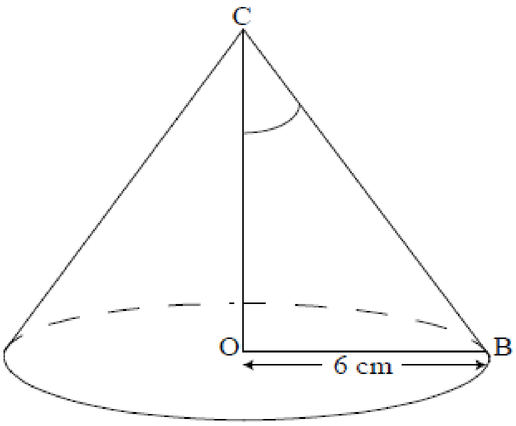
Find the height, OC, of the cone.

e. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



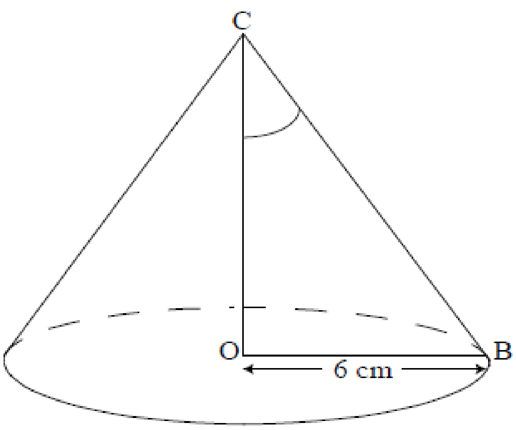
Find the size of angle BCO.

f. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



Find the slant height, CB.

g. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [4]



Find the total surface area of the cone.

Alex and Kris are riding their bicycles together along a bicycle trail and note the following distance markers at the given times.

Time (t hours)	1	2	3	4	5	6	7
Distance (d km)	57	65	72	81	89	97	107

a. Draw a scatter diagram of the data. Use 1 cm to represent 1 hour and 1 cm to represent 10 km. [3]

b.i. Write down for this set of data the mean time, $\bar{t}$. [1]

b.ii. Write down for this set of data the mean distance, $\bar{d}$. [1]

c. Mark and label the point $M(\bar{t}, \bar{d})$ on your scatter diagram. [2]

d. Draw the line of best fit on your scatter diagram. [2]

e. **Using your graph**, estimate the time when Alex and Kris pass the 85 km distance marker. Give your answer correct to **one decimal place**. [2]

f. Write down the equation of the regression line for the data given. [2]

g.i. **Using your equation** calculate the distance marker passed by the cyclists at 10.3 hours. [2]

g.ii. Is this estimate of the distance reliable? Give a reason for your answer. [2]

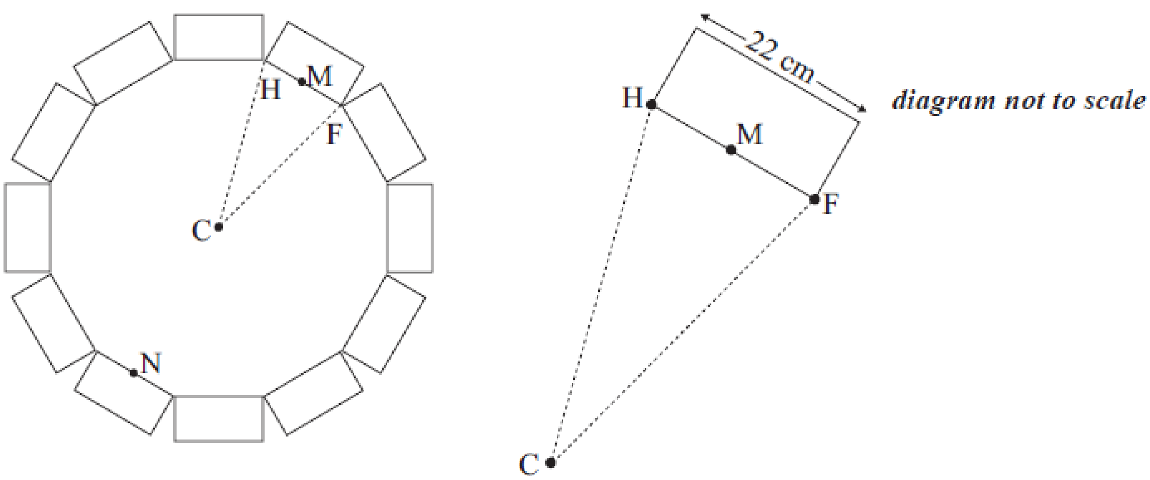
A gardener has to pave a rectangular area 15.4 metres long and 5.5 metres wide using rectangular bricks. The bricks are 22 cm long and 11 cm wide.

The gardener decides to have a triangular lawn ABC, instead of paving, in the middle of the rectangular area, as shown in the diagram below.

onbekend.png

The distance AB is 4 metres, AC is 6 metres and angle BAC is 40°.

In another garden, twelve of the same rectangular bricks are to be used to make an edge around a small garden bed as shown in the diagrams below. FH is the length of a brick and C is the centre of the garden bed. M and N are the midpoints of the long edges of the bricks on opposite sides of the garden bed.



The garden bed has an area of 5419 cm^2. It is covered with soil to a depth of 2.5 cm.

It is estimated that 1 kilogram of soil occupies 514 cm^3.

a.i. Calculate the total area to be paved. Give your answer in cm^2. [3]

a.ii. Write down the area of each brick. [1]

a.iii. Find how many bricks are required to pave the total area. [2]

b.i. Find the length of BC. [3]

b.ii. Hence write down the perimeter of the triangular lawn. [1]

b.iii. Calculate the area of the lawn. [2]

b.iv. Find the percentage of the rectangular area which is to be lawn. [3]

c.i. Find the angle FCH. [2]

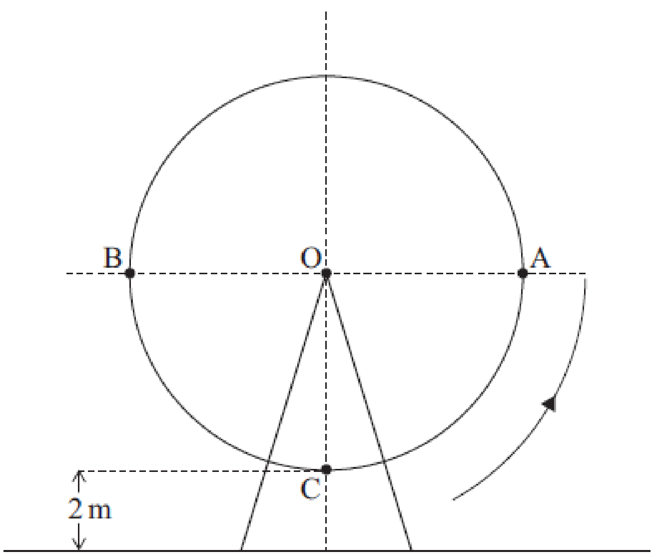
c.ii. Calculate the distance MN from one side of the garden bed to the other, passing through C. [3]

d. Find the volume of soil used. [2]

e. Find the number of kilograms of soil required for this garden bed. [2]

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.

diagram not to scale



a. Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction. [2]

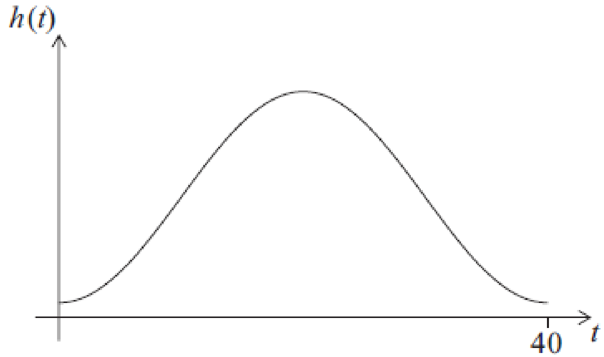
Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C .

b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . [2]

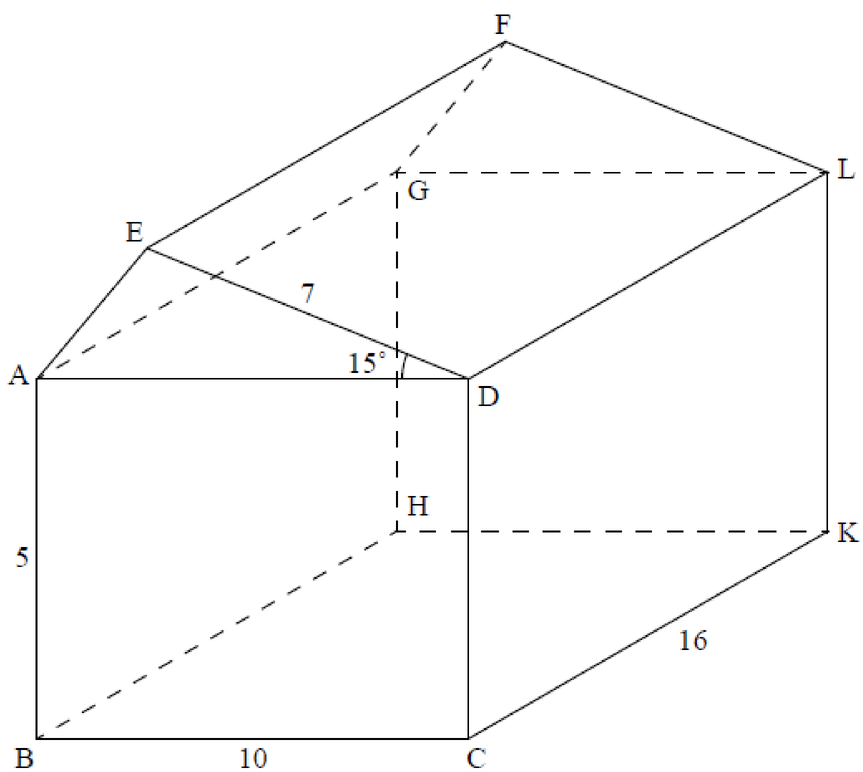
c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leqslant t \leqslant 40$. [4]



Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

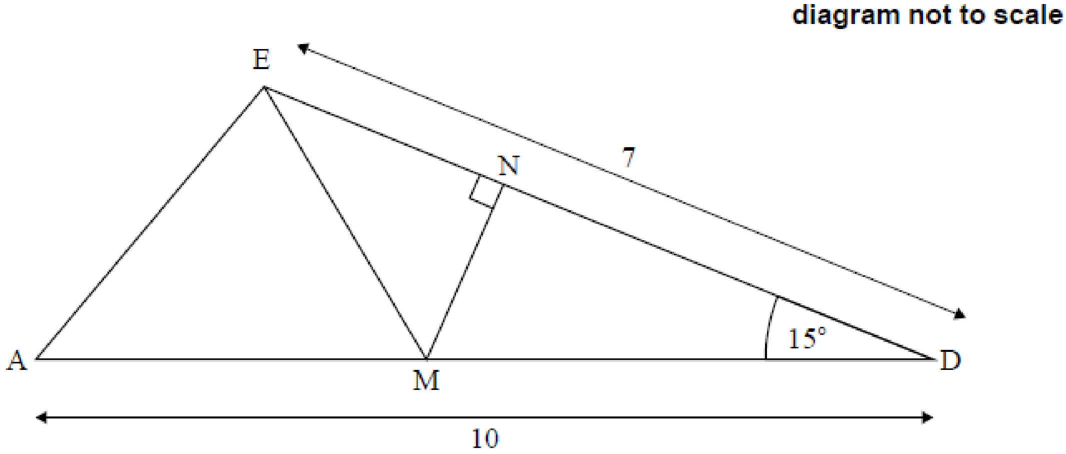
Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

diagram not to scale



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

Farmer Brown believes that N is the midpoint of ED.

a. Calculate the area of triangle EAD. [3]

b. Calculate the **total** volume of the barn. [3]

c. Calculate the length of MN. [2]

d. Calculate the length of AE. [3]

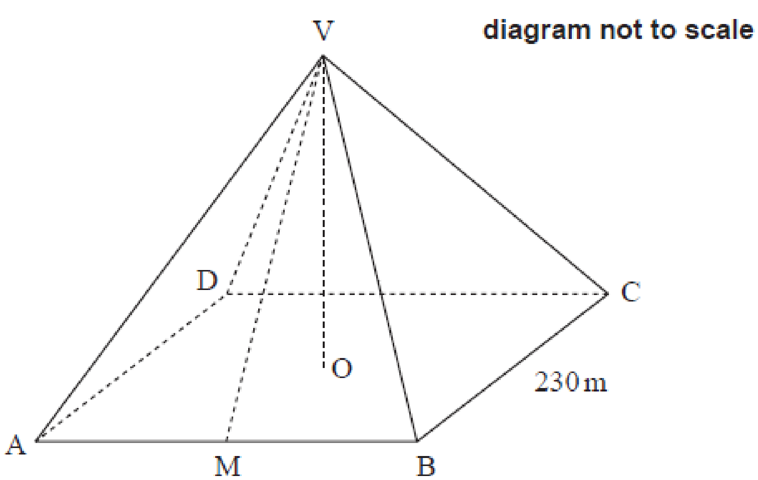
e. Show that Farmer Brown is incorrect. [3]

f. Calculate the **total** length of metal required for one support. [4]

---

a. The Great Pyramid of Giza in Egypt is a right pyramid with a square base. The pyramid is made of solid stone. The sides of the base are $230\,\mathrm{m}$ long. The diagram below represents this pyramid, labelled VABCD. [3]

V is the vertex of the pyramid. O is the centre of the base, ABCD . M is the midpoint of AB. Angle $\mathrm{ABV} = 58.3^{\circ}$.



Show that the length of VM is 186 metres, correct to three significant figures.

b. Calculate the height of the pyramid, VO . [2]

c. Find the volume of the pyramid. [2]

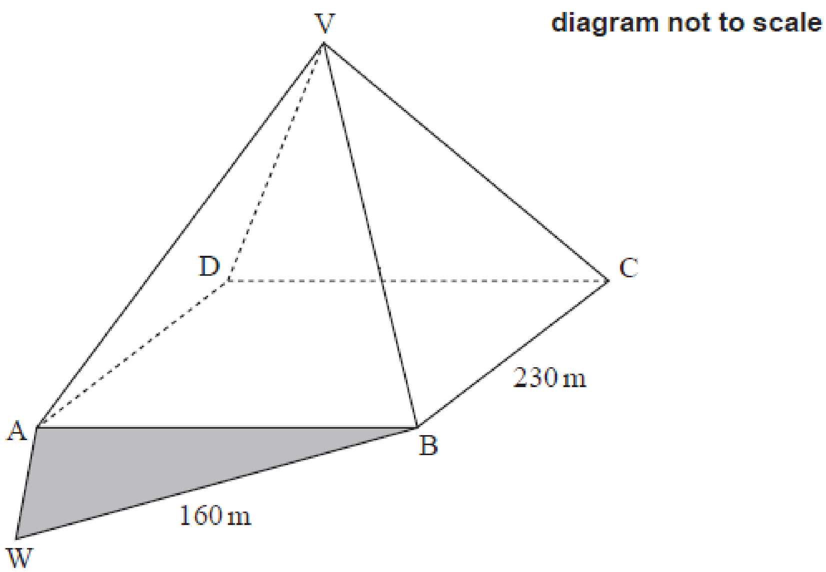
d. Write down your answer to part (c) in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Ahmad is a tour guide at the Great Pyramid of Giza. He claims that the amount of stone used to build the pyramid could build a wall 5 metres high and 1 metre wide stretching from Paris to Amsterdam, which are $430\,\mathrm{km}$ apart. [4]

Determine whether Ahmad's claim is correct. Give a reason.

f. Ahmad and his friends like to sit in the pyramid's shadow, ABW, to cool down. [6]

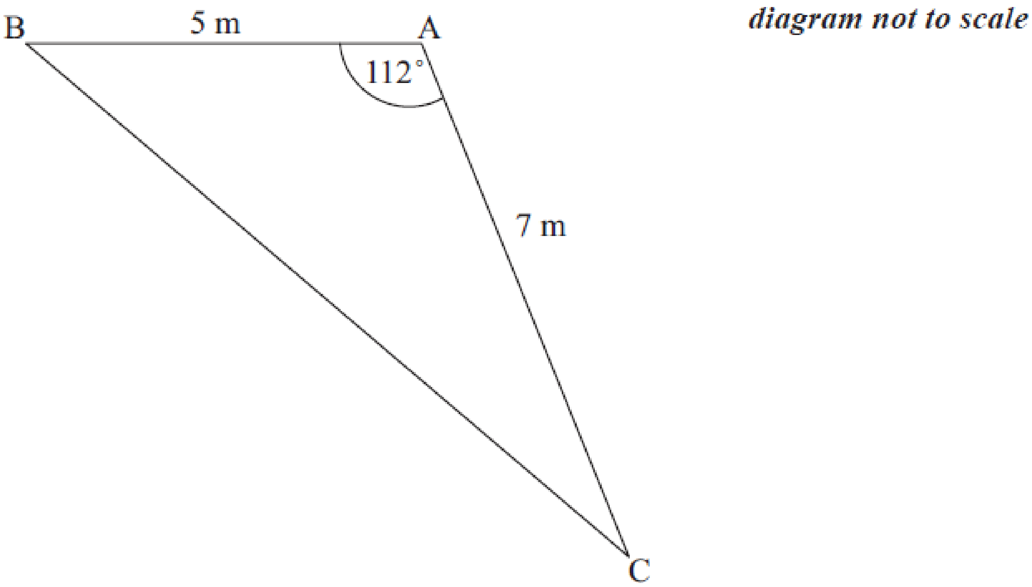
At mid-afternoon, $\mathrm{BW} = 160\,\mathrm{m}$ and angle $\mathrm{ABW} = 15°$.



i) Calculate the length of AW at mid-afternoon.

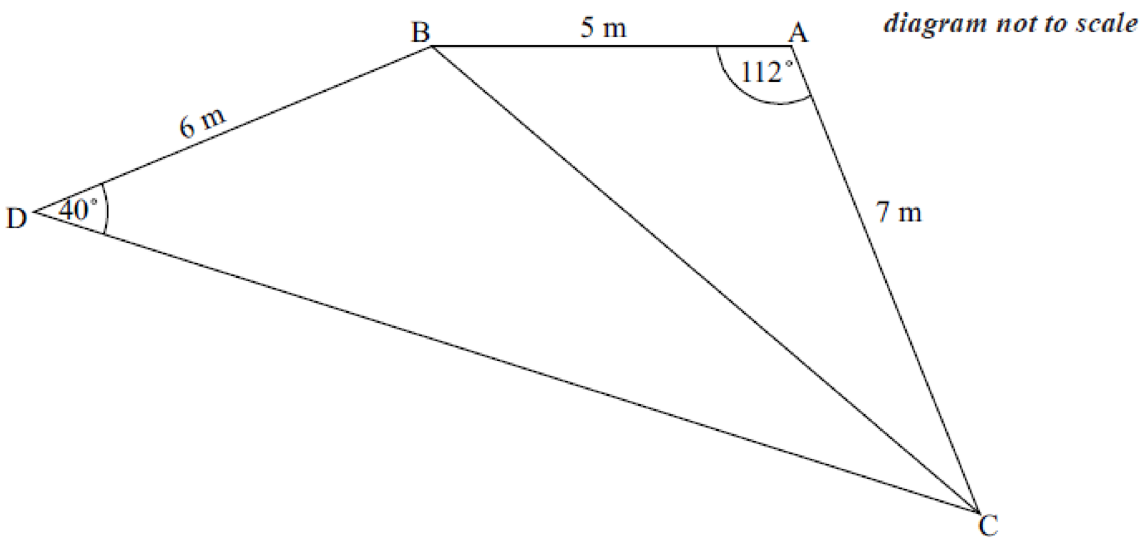
ii) Calculate the area of the shadow, ABW, at mid-afternoon.

A contractor is building a house. He first marks out three points A , B and C on the ground such that AB = 5 m , AC = 7 m and angle BAC = 112 °.



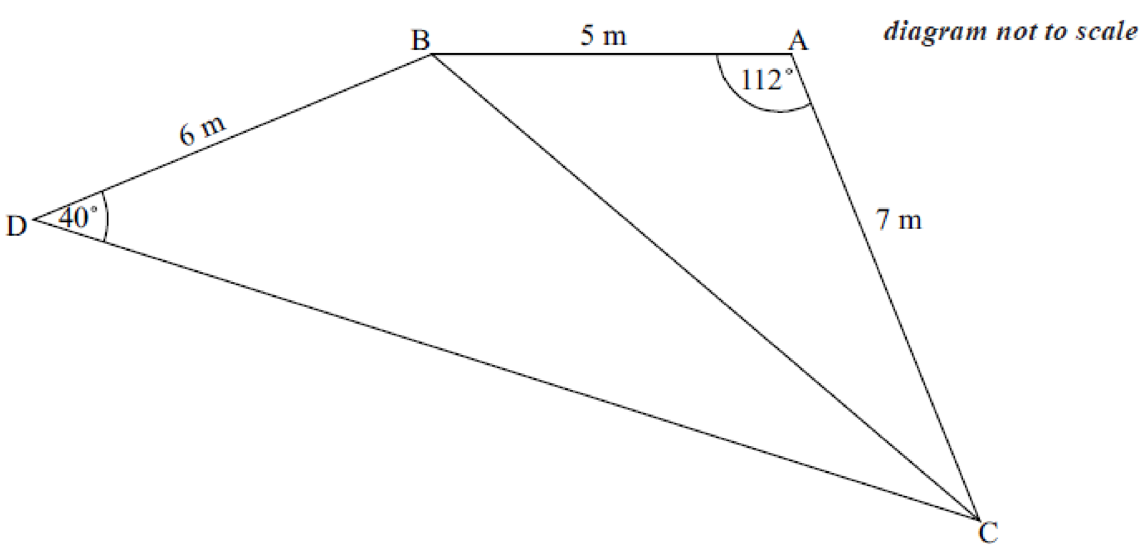
a. Find the length of BC. [3]

b. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



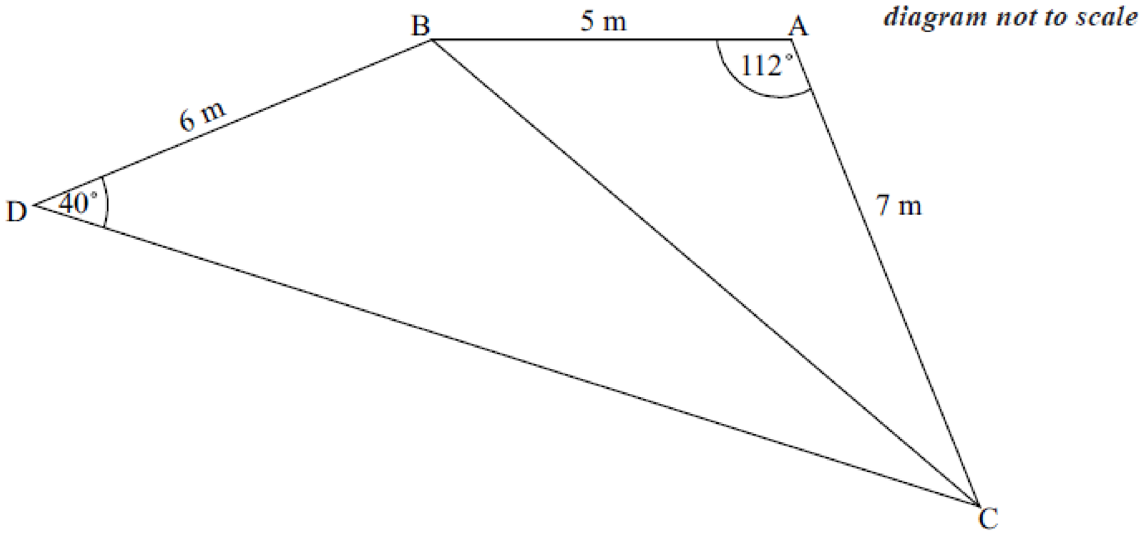
Find the size of angle DBC .

c. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



Find the area of the quadrilateral ABDC.

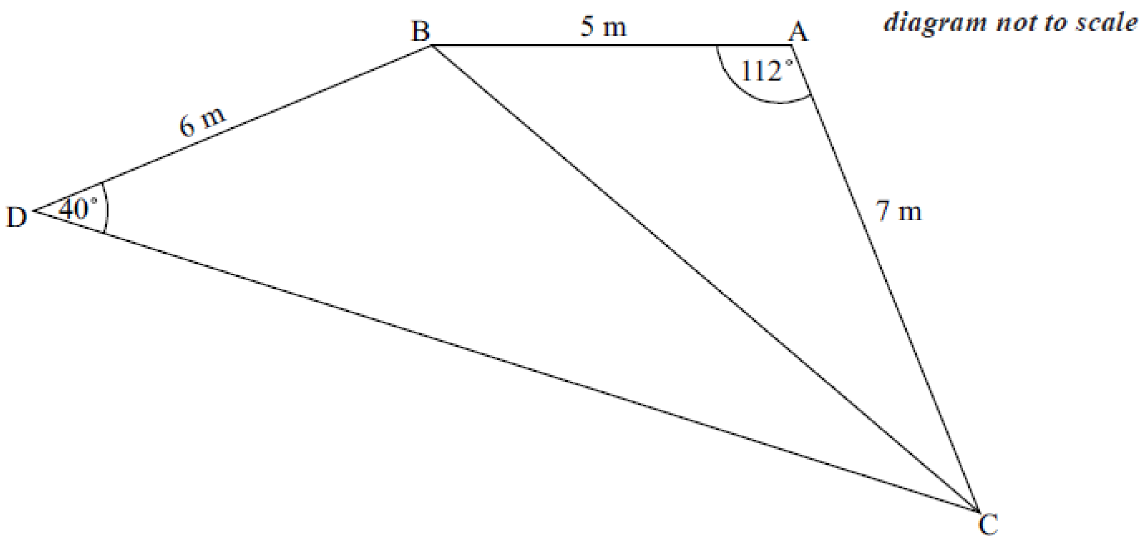
d. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [3]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

Find the volume of the soil removed. Give your answer in $\mathbf{m^3}$.

e. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [5]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

To transport the soil removed, the contractor uses cylindrical drums with a diameter of 30 cm and a height of 40 cm.

(i) Find the volume of a drum. Give your answer in $\mathbf{m^3}$.

(ii) Find the minimum number of drums required to transport the soil removed.

A random sample of 167 people who own mobile phones was used to collect data on the amount of time they spent per day using their phones.

The results are displayed in the table below.

Time spent per day (t minutes)	$0 \le t < 15$	$15 \le t < 30$	$30 \le t < 45$	$45 \le t < 60$	$60 \le t < 75$	$75 \le t < 90$
Number of people	21	32	35	41	27	11

Manuel conducts a survey on a random sample of 751 people to see which television programme type they watch most from the following:

Drama, Comedy, Film, News. The results are as follows.

	Drama	Comedy	Film	News
Males under 25	22	65	90	35
Males 25 and over	36	54	67	17
Females under 25	22	59	82	15
Females 25 and over	64	39	38	46

Manuel decides to ignore the ages and to test at the 5 % level of significance whether the most watched programme type is independent of **gender.**

i.a. State the modal group. [1]

i.b. Use your graphic display calculator to calculate approximate values of the mean and standard deviation of the time spent per day on these mobile phones. [3]

i.c. On graph paper, draw a fully labelled histogram to represent the data. [4]

ii.a. Draw a table with 2 rows and 4 columns of data so that Manuel can perform a chi-squared test. [3]

ii.b. State Manuel's null hypothesis and alternative hypothesis. [1]

ii.c. Find the expected frequency for the number of females who had 'Comedy' as their most-watched programme type. Give your answer to the nearest whole number. [2]

ii.d. Using your graphic display calculator, or otherwise, find the chi-squared statistic for Manuel's data. [3]

ii.e. (i) State the number of degrees of freedom available for this calculation. [3]

(ii) State his conclusion.

A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

15 like all three fruits

22 like mangoes and bananas

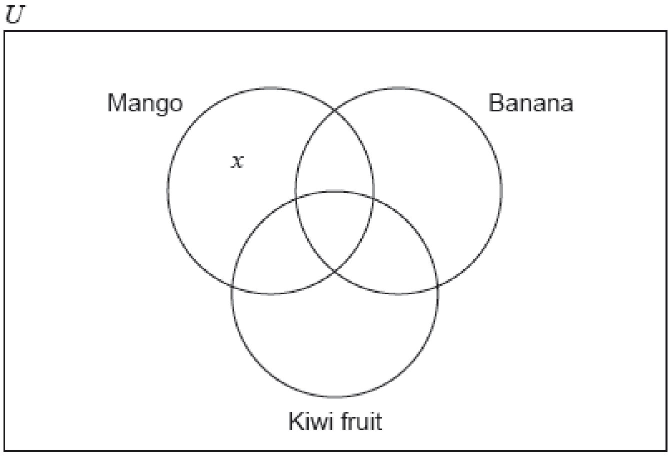
33 like mangoes and kiwi fruits

27 like bananas and kiwi fruits

8 like none of these three fruits

x like **only** mangoes

a. **Copy** the following Venn diagram and correctly insert all values from the above information. [3]



b. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [2]

Complete your Venn diagram from part (a) with this additional information **in terms of** x.

c. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [2]

Find the value of x.

d. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [2]

Write down the number of customers who like

(i) mangoes;

(ii) mangoes or bananas.

e. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [4]

A customer is chosen at random from the 100 customers. Find the probability that this customer

(i) likes none of the three fruits;

(ii) likes only two of the fruits;

(iii) likes all three fruits given that the customer likes mangoes and bananas.

f. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [3]

Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits.

A new café opened and during the first week their profit was \$60.

The café's profit increases by $10 every week.

A new tea-shop opened at the same time as the café. During the first week their profit was also $60.

The tea-shop's profit increases by 10 % every week.

a. Find the café's profit during the 11th week. [3]

b. Calculate the café's **total** profit for the first 12 weeks. [3]

c. Find the tea-shop's profit during the 11th week. [3]

d. Calculate the tea-shop's **total** profit for the first 12 weeks. [3]

e. In the mth week the tea-shop's **total** profit exceeds the café's **total** profit, for the first time since they both opened. [4]

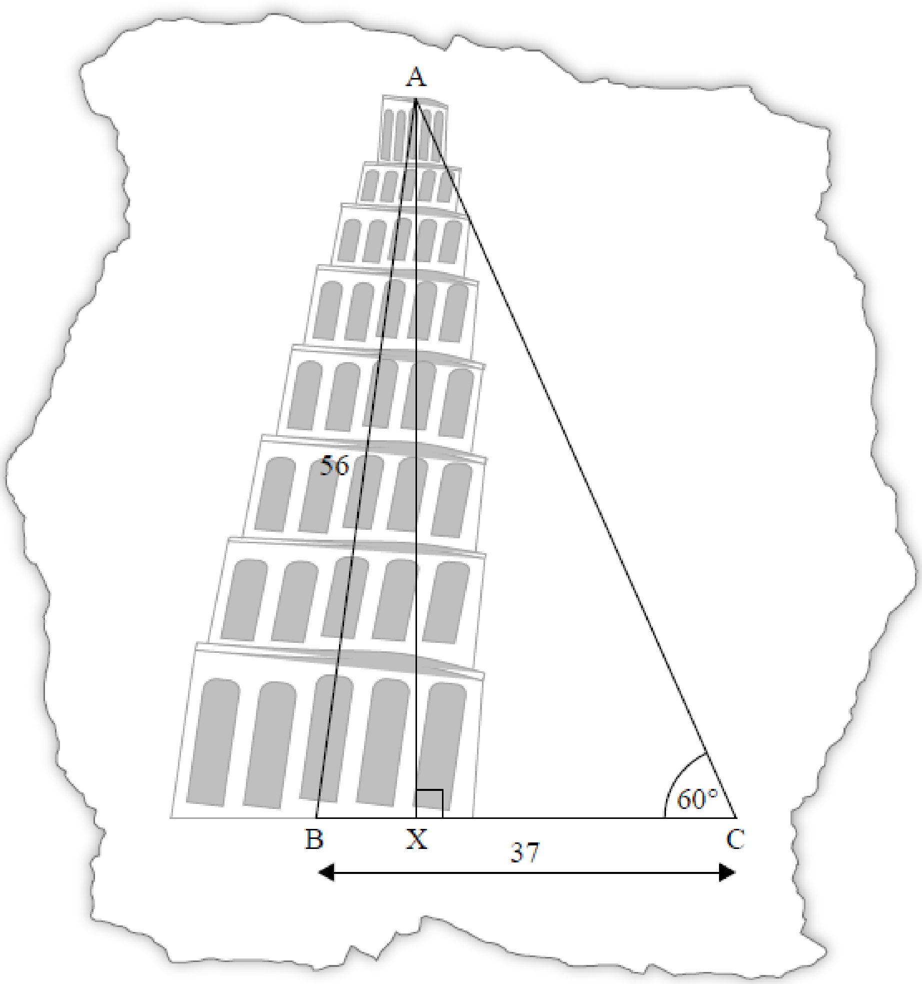
Find the value of m.

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



Giovanni's tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

a.i. Use Giovanni's diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. [5]

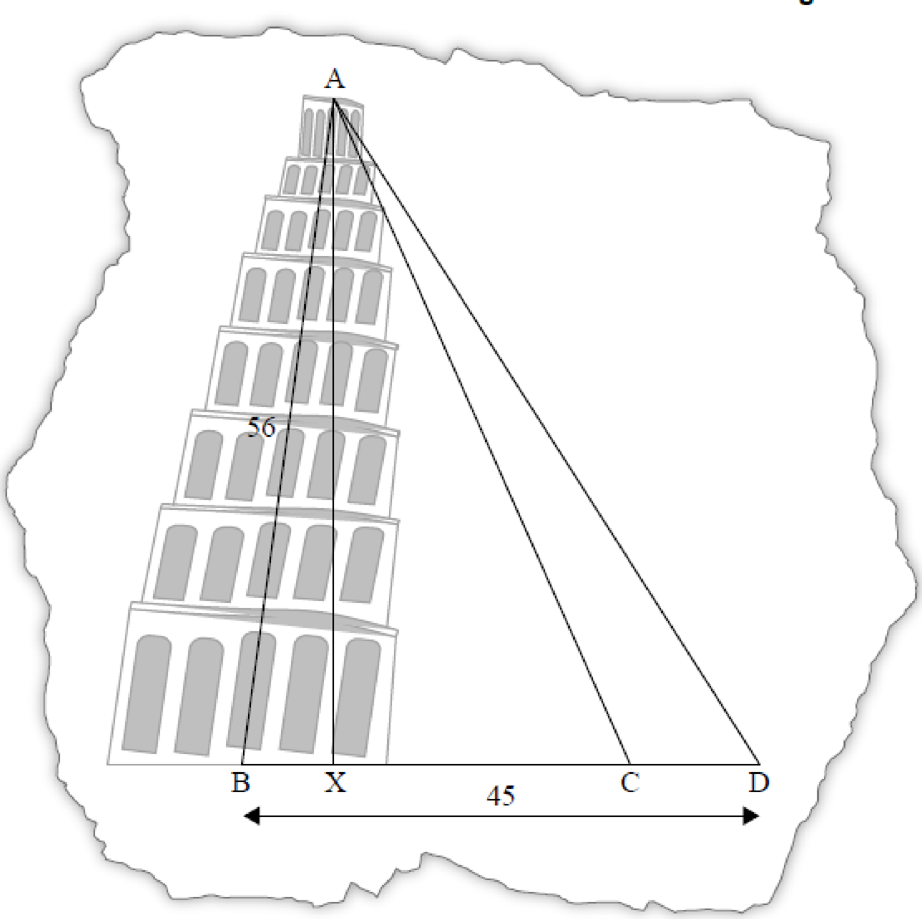
a.ii. Use Giovanni's diagram to calculate the length of AX. [2]

a.iii. Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. [2]

b. Find the percentage error on Giovanni's diagram. [2]

c. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. [3]

diagram not to scale



Find the angle of elevation of A from D.

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a. A distress flare is fired into the air from a ship at sea. The height, h, in metres, of the flare above sea level is modelled by the quadratic function [1]

$$h(t) = -0.2t^2 + 16t + 12\,,\ t \geqslant 0\,,$$

where t is the time, in seconds, and $t = 0$ at the moment the flare was fired.

Write down the height from which the flare was fired.

b. Find the height of the flare 15 seconds after it was fired. [2]

c. The flare fell into the sea k seconds after it was fired. [2]

Find the value of k .

d. Find $h'(t)$. [2]

e. i) Show that the flare reached its maximum height 40 seconds after being fired. [3]

ii) Calculate the maximum height reached by the flare.

f. The nearest coastguard can see the flare when its height is more than 40 metres above sea level. [3]

Determine the total length of time the flare can be seen by the coastguard.

George leaves a cup of hot coffee to cool and measures its temperature every minute. His results are shown in the table below.

Time, t (minutes)	0	1	2	3	4	5	6
Temperature, y (°C)	94	54	34	24	k	16.5	15.25

a. Write down the decrease in the temperature of the coffee [3]

(i) during the first minute (between t = 0 and t =1) ;

(ii) during the second minute;

(iii) during the third minute.

b. Assuming the pattern in the answers to part (a) continues, show that $k = 19$. [2]

c. Use the **seven** results in the table to draw a graph that shows how the temperature of the coffee changes during the first six minutes. [4]

Use a scale of 2 cm to represent 1 minute on the horizontal axis and 1 cm to represent 10 °C on the vertical axis.

d. The function that models the change in temperature of the coffee is $y = p\,(2^{-t}) + q$. [2]

(i) Use the values t = 0 and y = 94 to form an equation in p and q.

(ii) Use the values t =1 and y = 54 to form a second equation in p and q.

e. Solve the equations found in part (d) to find the value of p and the value of q. [2]

f. The graph of this function has a horizontal asymptote. [2]

Write down the equation of this asymptote.

g. George decides to model the change in temperature of the coffee with a linear function using correlation and linear regression. [4]

Use the **seven** results in the table to write down

(i) the correlation coefficient;

(ii) the equation of the regression line y on t.

h. Use the equation of the regression line to estimate the temperature of the coffee at t = 3. [2]

i. Find the percentage error in this estimate of the temperature of the coffee at t = 3. [2]

Consider the function $f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3$.

a. Sketch the graph of $y = f(x)$ for $-3 \le x \le 6$ and $-10 \le y \le 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the x-axis, and a scale of 1 cm to represent 1 unit on the y-axis. [4]

b. Find the value of $f(-1)$. [2]

c. Write down the coordinates of the y-intercept of the graph of $f(x)$. [1]

d. Find $f'(x)$. [3]

e. Show that $f'(-1) = -\frac{16}{3}$. [1]

f. Explain what $f'(-1)$ represents. [2]

g. Find the equation of the tangent to the graph of $f(x)$ at the point where x is -1. [2]

h. Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). [2]

i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [2]

Write down the value of

(i) a ;

(ii) b .

j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [1]

Describe the behaviour of $f(x)$ for $a < x < b$.

A biologist is studying the relationship between the number of chirps of the Snowy Tree cricket and the air temperature. He records the chirp rate, x, of a cricket, and the corresponding air temperature, T, in degrees Celsius.

The following table gives the recorded values.

Cricket's chirp rate, x, (chirps per minute)	20	40	60	80	100	120
Temperature, T (°C)	8.0	12.8	15.0	18.2	20.0	21.1

a. Draw the scatter diagram for the above data. Use a scale of 2 cm for 20 chirps on the horizontal axis and 2 cm for 4°C on the vertical axis. [4]

b. Use your graphic display calculator to write down the Pearson's product–moment correlation coefficient, r, between x and T. [2]

c. Interpret the relationship between x and T using your value of r. [2]

d. Use your graphic display calculator to write down the equation of the regression line T on x. Give the equation in the form $T = ax + b$. [2]

e. Calculate the air temperature when the cricket's chirp rate is 70. [2]

f. Given that $\bar{x} = 70$, draw the regression line T on x on your scatter diagram. [2]

g. A forest ranger uses her own formula for estimating the air temperature. She counts the number of chirps in 15 seconds, z, multiplies this number by 0.45 and then she adds 10. [1]

Write down the formula that the forest ranger uses for estimating the temperature, T.

Give the equation in the form $T = mz + n$.

h. A cricket makes 20 chirps in **15** seconds. [6]

For this chirp rate

(i) calculate an estimate for the temperature, T, **using the forest ranger's formula**;

(ii) determine the actual temperature recorded by the biologist, **using the table above**;

(iii) calculate the percentage error in the forest ranger's estimate for the temperature, compared to the actual temperature recorded by the biologist.